Call Protection, Financial Flexibility, and Debt Maturity Decisions*

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Abstract

This paper explores rollover risk management using callable bonds with call protection. Using a theoretical framework that allows firms to adjust debt refinancing timing within call periods, we analyze the trade-off between the benefits of increased timing flexibility and the increased costs of call risk demanded by bondholders. Our findings indicate that firms facing high rollover risks prefer shorter call protection lengths. This strategy enhances creditworthiness and promotes earlier refinancing calls that align closely with the start of call periods. Consequently, call protection length emerges as a more accurate indicator of effective maturity. Our empirical evidence strongly supports these dynamics.

(JEL Classification: G3, G12, G32, G33)

Keywords: stated maturity, effective maturity, callable bond, call protection, rollover risk

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1 Introduction

Research on corporate debt maturity underscores the critical role that strategic debt profile management plays in mitigating rollover risk, as highlighted by CFO surveys conducted by Graham and Harvey (2001) and Servaes and Tufano (2006). These surveys identified rollover risk as a key factor in shaping debt maturity structures and underscored the need to understand more clearly how firms actively manage their debt maturities. Choi et al. (2018) addressed this issue by showcasing the variety of strategies that firms employ to address rollover risk through adjustments in debt maturity. Motivated by these insights, we use this paper to analyze the novel role that refinancing callable bonds plays in debt maturity management. Firms use callable bonds strategically to reconfigure their debt structures, which allows for the early redemption and replacement of existing bonds. This approach not only introduces a significant layer of flexibility in managing debt maturities but also highlights that effective maturity serves as a pivotal element in firms' strategy to mitigate rollover risk. Despite its importance, the strategic use of callable bonds and its impact on effective maturity and rollover risk management has not been thoroughly examined. Our research aims to fill this void by analyzing debt maturity management strategies more fully, as well as addressing their implications for financial stability and risk mitigation.

As a motivating example, the U.S. corporate bond market, where nearly 90% of all bonds feature call provisions, presents a notable shift from the conventional focus on nominal debt maturity stated at issuance (abbreviated as stated maturity, hereafter).[1] These provisions allow firms to issue long-term bonds with the option for early redemption, thus offering the flexibility to effectively shorten a bond's term. Figure 1, Panel A, shows a marked divergence between the effective and stated maturities of bonds, and

[1]According to statistics reported by the Securities Industry and Financial Markets Association, the corporate bond issuance volume in the US market grew from 337.4 billion in 1996 to 2,274.9 billion dollars in 2020; bonds with call provisions also increased from 14% of the total issuance volume in 1996 to nearly 90% in 2020.

captures a growing tendency for effective maturity to mirror call protection periods.[2] This pattern has become increasingly apparent over the past three decades, thus making call protection periods an essential, though often overlooked, measure of a bond's real lifespan.[3] Firms are adopting strategies that prioritize retiring callable bonds just as call protection expires, underscoring the significance of call protection duration in defining true maturity. This evolution towards incorporating call protection as a key factor in maturity planning highlights a critical area for research: developing a comprehensive framework for choosing call protection lengths that effectively inform debt maturity decisions.

Our study addresses a significant gap in the finance literature by introducing a theoretical model that determines the optimal length of call protection for callable bonds. We build upon the structural credit risk framework by Leland and Toft (1996), which links debt default decisions to stated maturities and refinancing (rollover) commitments. Our innovative model considers a debt structure that includes both non-callable and longer-term callable bonds with lumpy maturities – a feature prevalent in financial data (Choi et al., 2018) and the corporate bond market (as shown in Figure 1, Panel B).[4] We also explore how call protection, a critical but often neglected aspect in existing models,[5] strategically provides firms with a certain level of flexibility, so they may refinance early to benefit their shareholders.[6] To identify the best refinancing strategies after the call

[2]This discrepancy is likely attributed to the fact that callable bonds are almost twice as likely to be retired early as non-callable bonds (Brown and Powers, 2020). Generally, non-callable bonds can only be redeemed early through tender offers or repurchases at their prevailing market prices, whereas callable bonds can be redeemed early through tender offers, repurchases, or calls. In particular, predetermined call prices for fixed-price callable bonds establish upper limits on redemption prices, thereby facilitating early redemption compared to redemption through make-whole calls. On the other hand, a make-whole call price, which is determined *ex post* according to the prevailing level of interest rates, usually serves as the cap on the price of a successful tender offer (Mann and Powers, 2003). Therefore, the presence of a make-whole call provision can alleviate uncertainty of the early redemption through a tender offer (Brown and Powers, 2020).

[3]The decline in the average stated maturity of newly-issued corporate bonds is first documented by Custódio et al. (2013). This trend is driven not only by sell-side factors in the corporate bond market but also by buy-side ones (Greenwood et al., 2010; Paligorova and Santos, 2017; Butler et al., 2022).

[4]Without the *ex ante* commitment to a bond's stated maturity, He and Milbradt (2016) and Hu et al. (2023) study a firm's ability to adjust stated maturity over time through the debt structure involving non-callable bonds with different stated maturities.

[5]For example, see Leland (1998), Goldstein et al. (2001), Titman and Tsyplakov (2007), Chen (2010), Morellec et al. (2012), Chen et al. (2021), Dangl and Zechner (2021), among others.

[6]Our framework focuses on call-driven early refinancing activities, as in Chen et al. (2021) and Dangl

protection period, our model applies a backward-recursive pricing algorithm through a tree-based "forest" approach (Liu et al., 2016).

A key feature in our model is that it enables issuers of callable bonds to set a call period between the first call date and a bond's stated maturity date, rather than committing to a single maturity date at issuance. Issuers can then tailor a bond's effective maturity with early refinancing, choosing the most opportune moment based on their financial health. This capability prevents the need to issue new bonds during times of financial strain, which thus protects the issuer's credit quality.

Our model produces four key outcomes, starting with the determination of the optimal call protection length. By setting a bond's stated maturity, a shorter call protection period creates more opportunities for early refinancing, thereby providing issuers with greater flexibility that helps prevent debt defaults amid significant rollover risks. However, while this reduction in a call protection period boosts a firm's credit standing and lowers the default risk premium that bondholders demand, it simultaneously raises the call risk premium. Consequently, firms must balance the benefits of increased timing flexibility against the increased costs associated with call risk. Our theoretical framework pinpoints the optimal call protection length that maximizes the total value of a levered firm at inception. Based on this benefit-cost trade-off, our calibrated model recommends a realistic 3-year call protection for a 10-year callable bond, which is consistent with empirical observations.

Second, our model explores how the timing flexibility provided by predetermined call periods affects a firm's early refinancing decisions as well as the effective maturities of its callable bonds. By strategically determining call protection lengths, firms can adjust refinancing times post-issuance based on their financial status. This flexibility helps firms avoid issuing new bonds during financial downturns, thus minimizing bankruptcy costs

and Zechner (2021), so we may directly associate refinancing timing with the stated call protection period. In the model of DeMarzo and He (2021), early refinancing is, on the other hand, driven by a debt repurchase at the prevailing debt market price.

linked to rollover risks and enhancing overall levered firm value. This increased value encourages firms to initiate refinancing earlier within call periods. Through proactive debt repricing and refinancing, firms secure gains for shareholders, reducing the potential for risk-shifting or underinvestment behaviors (Myers, 1977; Lambrecht and Myers, 2008; Diamond and He, 2014).[7] As a result, this "call-to-shorten" strategy shifts the effective maturity dates of callable bonds towards the start of call periods, making call protection length a more accurate measure of a bond's real lifespan. Our calibrated model quantitatively predicts an effective maturity of about 4.2 years for a 10-year callable bond with a 3-year call protection, which closely aligns with empirical data.

Third, we analyze how firms with varying levels of leverage select their call protection lengths and their timing of early refinancing. High-leverage firms, those which face greater rollover risks compared to their low-leverage peers (Childs et al., 2005; He and Xiong, 2012b), gain more by shortening call protection periods to increase the timing flexibility of call provisions. Our model suggests that these firms are inclined to reduce call protection lengths, so they may better handle rollover risks. This preference is also evident in firms dependent on short-term debt, which typically undergo frequent rollovers. Moreover, the strategic determination of call protection lengths allows for call-to-shorten refinancing strategy to be adopted, which closely aligns effective maturities of callable bonds with their call protection periods. Consequently, our model predicts a marked disparity between effective and stated maturities as firms choose shorter call protection periods.[8] These insights clarify the strategic decision-making between non-callable and longer-term callable bonds, both of which can mitigate conflicts between shareholders and bondholders (Robbins and Schatzberg, 1986). Hence, our model indicates that firms

[7] More specifically, by keeping debt structure flexible to avoid substantial rollover risk, issuers of callable bonds can effectively enhance creditworthiness and thus alleviate the underinvestment problem stemming from debt overhang (Myers, 1977, 1984).

[8] Samet and Obay (2014) empirically examine call risk premiums in a global framework, finding that firms with high leverage have a higher call risk premium than those with low leverage. Our model rationalizes this finding by demonstrating that high-leverage firms tend to choose shorter call protection periods and redeem their callable bonds earlier.

with higher rollover risk exposure might simulate short-term borrowing by issuing callable bonds with brief call protection periods.

Table 1 provides two illustrations to demonstrate how two high-leverage firms execute this simulation. In Panel A, General Mills issued 25-year and 12-year callable bonds in 1998 and 1999, with 5-year and 4-year call protections, respectively. Early redemption transformed these into effectively shorter-term bonds. General Mills then issued four callable bonds with 1-year call protections, simulating short-term borrowing. Panel B meanwhile shows Barclays Bank issuing three callable bonds with 1-year call protections in 2011, redeemed early to act as 1-year bonds. Barclays continued this approach by issuing two more callable bonds with 1-year protections, perpetuating the strategy of emulating short-term borrowing.

Lastly, we explore the welfare implications of strategically utilizing callable bonds. By comparing the interest costs of refinancing with callable bonds to those of rolling over shorter-term non-callable bonds, we assess the financial benefits of callable bonds. This comparison, based on data trends over the past two decades depicted in Figure 1, matches the stated maturities of non-callable bonds with the call protection lengths of callable bonds, so we may evaluate the cost-effectiveness of specifying a call period rather than a single maturity date at issuance.

While callable bonds may initially seem more expensive due to call risk premiums – especially for low-leverage firms with minimal rollover risks – the scenario shifts for high-leverage firms. For the latter, shorter call protection periods increase timing flexibility, reduce the risk of financial distress, and lower interest costs, thereby making callable bonds a more economical option by reducing default risk premiums. This flexibility is particularly crucial for firms with significant rollover risk exposure, supporting the notion that financial flexibility enables cost-effective financing solutions (Gamba and Triantis, 2008). Research consistently shows that firms with higher leverage or those dependent on short-term debt tend to favor callable bonds (Chen et al., 2010; Brown and Powers, 2020;

Cathcart et al., 2020).

We validate our model's predictions with empirical data, which robustly supports our theoretical insights. Our analysis highlights three key findings: firms' preference for shorter call protection periods, the divergence of effective maturities from stated maturities, and the decrease in interest costs achieved through the strategic use of callable bonds. Specifically, firms facing higher rollover risks consistently choose shorter call protections, engage in earlier debt refinancing, and align effective maturities more closely with call protection durations. Our data indicate that for 10-year callable bonds, those issued by high-leverage firms feature call protection periods that are, on average, 1.3 years shorter than those issued by low-leverage firms. Furthermore, high-leverage firms tend to redeem these bonds approximately 1.1 years earlier than their low-leverage counterparts. This behavior results in a notable 3.02% decrease in call protection lengths relative to stated maturities through early refinancing over time, compared to just a 0.95% reduction in low-leverage firms. Additionally, firms with intense refinancing needs tend to exercise call options sooner, with an average reduction in effective maturity of about 18%, which contrasts sharply with the 7% reduction observed in firms with less frequent refinancing needs. This empirical evidence highlights a clear trend toward shorter call protection and earlier refinancing among firms with heightened rollover risks.

Finally, we analyze how shorter call protection and earlier refinancing affect firms' interest costs on their outstanding bonds. Despite expectations that shorter call protection could increase call risk premiums, our findings indicate that the average coupon rate remains stable or even decreases after refinancing. This finding highlights the significant advantages of the financial flexibility that callable bonds provide, especially in helping to lower default risk premiums. These results demonstrate that proactive debt management effectively reduces rollover risks without raising costs.

This paper makes several contributions to the literature. Building upon insights from Powers (2021), we enhance the discourse on call protection by presenting a novel theoretical model used to identify the optimal call protection length, which marks a significant advance in this area. Previously, Marr and Ogden (1989) uniquely connected the length of call protection to a trade-off between benefits and costs, noting that while shorter call protection periods increase flexibility to address underinvestment (Thatcher, 1985) and information asymmetry issues (Robbins and Schatzberg, 1986), they also necessitate higher call risk premiums. Their research indicated that firms facing significant risks related to these issues often opt for shorter call protection periods.

Our model extends this argument by linking the choice of call protection length to the intricate balance between rollover risk and the debt overhang problem, which leads to underinvestment (Myers, 1977, 1984). As we consider the timing flexibility that a call period provides, our approach evaluates this balance through the lens of bondholders' required call risk and default risk premiums related to rollover risk. As noted by Powers (2021), the optimal call protection length derived from our model mirrors the considerable variations observed in practice, offering a comprehensive view of how firms navigate these complex trade-offs.

Secondly, our paper contributes to the extensive literature on rollover risk management, which explores how firms use financial flexibility to overcome challenges that might hinder refinancing efforts. Such strategies include adjusting strategic cash reserves (Bolton et al., 2011; Acharya et al., 2012; Harford et al., 2014) and modifying leverage (Childs et al., 2005; DeMarzo and He, 2021; Dangl and Zechner, 2021). Brunnermeier and Yogo (2009) highlight the use of short-term borrowing to allow quicker debt maturity adjustments, thereby improving the prospects of securing long-term financing before economic downturns. Similarly, He and Milbradt (2016) and Hu et al. (2023) explore the strategic use of both short-term and long-term non-callable debt, analyzing how firms adjust the proportions of each type in response to worsening credit conditions.

Choi et al. (2018, 2021) analyze the strategic management of debt maturity dispersion, exploring how firms can mitigate rollover risks by diversifying the distribution of their stated maturities. Conversely, Mian and Santos (2018) focus on how firms adjust the timing of their debt refinancing, finding that creditworthy firms are more inclined to refinance early under favorable credit conditions, so they may secure longer loan maturities as a hedge against future borrowing under less favorable conditions. Studying the corporate bond market, Xu (2018) and Ma et al. (2023) demonstrate that speculative firms frequently use callable bonds to refinance earlier than planned, thereby lengthening the average stated maturity of their outstanding bonds.

Our paper contributes to this literature by providing insights on the timing flexibility offered by the call period. Although Xu (2018) and Ma et al. (2023) acknowledge the use of call provisions for early refinancing, they view call protection length as an externally determined term based on the rule of thumb. Our key contribution is that we incorporate the choice of call protection length into the dynamic trade-off framework of Leland and Toft (1996), which interrelates debt default decisions with rollover risk. This incorporation not only allows us to decode callable bonds from the broader credit view (Becker et al., 2024) but also to explain significant variations in protection length observed in practice (Powers, 2021). In addition, a callable bond with call protection embodies the unique attributes of both short-term and long-term non-callables, and its multiple scheduled call dates within a call period further allows firms to secure long-term financing from the outset while retaining the option to refinance early at their discretion after issuance. This makes callable bonds a more adaptable and effective instrument for managing rollover risk compared to that of solely relying on a combination of non-callables, as discussed in He and Milbradt (2016) and Hu et al. (2023). Moreover, the strategy of tailoring call protection lengths and call date numbers closely resembles the strategy of dispersing debt maturity to reduce rollover risk. This resemblance allows our findings to align with insights of Choi et al. (2018, 2021), which demonstrate that firms with higher leverage prefer to adopt more dispersed maturity structures. Similarly, we reveal that such firms tend to opt for shorter call protection periods to increase call date numbers.

Third, our paper contributes to the discussion on how financial flexibility affects debt maturity decisions. Traditionally, firms with higher leverage that can only set one maturity date at issuance might extend the term to maturity to mitigate rollover risk (Diamond, 1991).[9] Financial flexibility significantly changes this scenario however, allowing firms to hedge against rollover risk more effectively, which in turn lowers bankruptcy costs and enhances the value of levered firms. Childs et al. (2005) and DeMarzo and He (2021) explain that the ability to adjust leverage favors the selection of short-term bonds, as more frequent debt repricing and refinancing help retain value increases for shareholders. Similarly, our analysis of callable bonds shows that the timing flexibility provided by predetermined call periods encourages earlier refinancing and positions effective maturity dates earlier in these periods. This insight aligns with the dynamics depicted in Figure 1, Panel A, indicating that call protection lengths are a more precise measure of a bond's effective maturity.

Lastly, our paper contributes to the discourse on the widespread use of callable bonds (Chen et al., 2010; Booth et al., 2014; Becker et al., 2024) as we compare them with short-term non-callable bonds. Robbins and Schatzberg (1986) suggest that short-term non-callable and long-term callable bonds can act as perfect substitutes to manage conflicts between shareholders and bondholders. However, callable bonds provide distinct advantages that enhance their appeal. Firstly, they simplify fundraising by appealing to major investors like insurance companies that prefer long-term bonds to align with their liabilities and reduce exposure to interest rate risks (Butler et al., 2022). Secondly, our model underscores that callable bonds offer long-term financing with the flexibility of early refinancing, enabling issuers to manage agency conflicts effectively without the significant risks associated with rollover.[10]

[9]This positive relationship between firm leverage and stated maturity is empirically confirmed by several earlier studies, such as Barclay and Smith (1995), Stohs and Mauer (1996), Johnson (2003), and MacKay (2003).

[10]This implies that insurance companies should pay more attention to call protection length rather

The rest of the paper is structured as follows. In Section 2, we introduce our theoretical framework for investigating the optimal choice of call protection length, as well as introduce our backward-recursive pricing algorithm grounded in the tree-based method. In Section 3, we describe our bond-level and firm-level data and elaborate upon our estimation procedure for the variables that characterize these datasets. Additionally, we also outline how our model parameters are calibrated by using these data. Next, using our calibrated model, we analyze the choice of call protection length and how debt refinancing timing relates to this choice in Section 4. Moreover, we study the welfare implication of strategically using callable bonds by comparing interest costs of refinancing with callable bonds to those of rolling over shorter-term non-callable bonds. After we discuss our evidence on the predictions derived from our model in Section 5, we conclude in Section 6.

2 Model

We build our theoretical framework on the structural credit risk model by Leland and Toft (1996),[11] incorporating a connection between endogenous default and early refinancing actions triggered by callable bonds. Specifically, a firm can only announce a call after a pre-established call protection period expires. Due to analytical challenges posed by the free boundary problem in valuing debt and equity within this context, we develop a backward-recursive tree-based algorithm. This algorithm, which expands upon the state-transition forest concept introduced by Liu et al. (2016), allows us to numerically address valuation complexities.

Our structural model setup, which adopts a more nuanced approach to debt maturity to reflect real-world lumpiness, is outlined in Section 2.1. Section 2.2 then delves into a

than the stated maturities of callable bonds when constructing their portfolios, since bonds' real lifespan tends to be closer to the protection length. The choice of callable bonds with short call protection periods helps alleviate issuers' rollover risk as well as raises investors' reinvestment risk.

[11]This model setting is generic and applies to both financial and non-financial firms, as in He and Xiong (2012b) and Della Seta et al. (2020).

firm's default and (early) refinancing decisions, including how a firm selects call protection length. We outline the construction of our forest structure in Section 2.3, providing a detailed procedural explanation in Appendix A.1, as well as a robustness check.

2.1 Contingent Claims under a Structural Model with Lumpy Debt Maturity

A structural model specifies the evolution of a firm's asset value and treats equity and debt as contingent claims on a firm's assets. We follow He and Xiong (2012b) by supposing that the asset value of an all-equity firm at time t, V_t, follows a lognormal diffusion process under the risk-neutral probability measure:

$$\frac{dV_t}{V_t} = (r - q)dt + \sigma dz, \tag{1}$$

in which the non-negative constant r and q denote the interest rate and the payout ratio, respectively. The firm uses its available cash flow at time t, $qV_t dt$, to first fulfill contractually-obligated debt payments, and the remaining value (if any) is distributed to the firm's shareholders as dividends.[12] If the cash payout cannot fulfill the debt obligation, then the firm will either issue new equity to cover the deficit or announce default as the deficit surpasses the equity value (Chen, 2010). The firm value volatility σ reflects the firm's business risk (Merton, 1974) and is set to a positive constant, as Fan and Sundaresan (2000) argue that a firm (or its shareholders) cannot alter business risk arbitrarily due to the presence of restrictive bond covenants. dz denotes a standard Brownian motion, which represents random shocks to a firm's fundamental. As in He and Xiong (2012b), a firm's fundamental is reflected by V_t.

When modeling contractually-obligated debt payments on a bond with a finite ma-

[12] This setting implies that the firm does not hold cash reserves, as in Chen (2010). It enables us to eliminate the influence of the flexibility in adjusting cash holdings for hedging against rollover risk. By keeping more cash, a firm can mitigate its exposure to rollover risk due to market illiquidity (Bolton et al., 2011; Harford et al., 2014).

turity, many existing theoretical frameworks, such as He and Xiong (2012b) and Dangl and Zechner (2021), assume that the bond's principal is amortized at a constant rate over time. Instead, our framework follows Geelen (2016) and Chen et al. (2021) in considering the setting of lumpy debt maturity, which signifies that the principal will simultaneously mature on the stated maturity date. This feature is crucial in relating observed bond maturities with associated default risk due to its prevalence in real-world observations (Choi et al., 2018).

We consider the capital structure composed of equity, a T-year callable bond with a P-year call protection ($P \in (0, T]$), and a shorter-term T/m-year non-callable bond ($m > 1$), denoted by E^c, CB^c, and SB^c, respectively. The superscript "c" indicates the inclusion of a callable bond in the capital structure. Their values, on the other hand, are denoted by $E^c(V_t, t)$, $CB^c(V_t, t \,|\, P, T)$, and $SB^c(V_t, t \,|\, T/m)$, for which the parameters following the vertical bar represent the lengths of bond maturities and the call protection, respectively, as determined right before their issuance. The firm can only call the CB^c back at time t after the expiration of the call protection (i,e. $t \in [P, T]$) using the effective call price equal to the scheduled call price K_t plus accrued interest. Typically, K_t is set to the bond face value plus a call premium equal to a certain percentage of one year's interest payment (Tewari et al., 2015; Powers, 2021). We let F_L (F_S) be the face value, and C_L (C_S) be the continuous coupon rate for the CB^c (SB^c). The total levered firm value at $t = 0$, $V_0^{L.c}$, is then expressed as:

$$V_0^{L.c} = E^c(V_0, 0) + SB^c(V_0, 0 \,|\, T/m) + CB^c(V_0, 0 \,|\, P, T). \tag{2}$$

A callable bond grants its issuer the flexibility to adjust debt refinancing timing. To distinguish between refinancing scenarios that allow for timing flexibility and those that do not, we compare the interest costs of refinancing with callable bonds to those of rolling over shorter-term non-callable bonds. We consider another firm with a two-bond debt structure composed of P-year and T/m-year non-callable bonds. The firm's total levered firm value at $t = 0$, $V_0^{L.s}$, is expressed as:

$$V_0^{L.s} = E^s(V_0, 0) + SB^s(V_0, 0 \,|\, T/m) + SB^s(V_0, 0 \,|\, P), \tag{3}$$

in which the superscript "s" is attached to contingent claims in the capital structure without any callable bond when they are initially issued. T/m-year SB^s is identical to SB^c in Equation (2) in all other aspects. We align the stated maturity of another SB^s with call protection length P of the CB^c in light of our observation from Figure 1. This figure shows that over the past two decades, the average call protection length (depicted in blue in Panel A) is close to the average stated maturity of non-callable bonds (shown in cyan in Panel B). For the purposes of this paper, we pinpoint two targets. First, we aim to quantify the impact of stating a call period rather than just a single maturity date upon issuance by comparing the coupon rate of CB^c in Equation (2) with that of the P-year SB^s in Equation (3). Second, we aim to examine how this impact influences other bonds in the same debt structure by comparing the difference in coupon rates between the two otherwise identical T/m-year SB^c and SB^s.

In our analysis, we consider three types of market frictions as in Childs et al. (2005). First, costs for raising new bonds are assumed to be γ proportion of the bonds' market value, in which $\gamma \in (0, 1)$.[13] When employing bonds, the firm earns tax shield benefits while also facing bankruptcy costs. When the firm remains solvent, its coupon payments benefit from a tax-deductible rate τ, in which $\tau \in (0, 1)$. However, when the firm goes bankrupt and undergoes liquidation, a fixed proportion ω of the firm's asset value V_t is forfeited as liquidation expenses, in which $\omega \in (0, 1)$.

[13]For simplicity's sake, we do not consider equity issuance costs when new equity is issued to cover debt obligation, as in He and Xiong (2012b) and Dangl and Zechner (2021).

2.2 Decision on Default, Early Refinancing, and the Call Protection Length

To ensure a fair comparative analysis between refinancing using callable bonds and using shorter-term non-callable bonds, we assume that a firm employs a debt structure with two bonds of equal seniority and follows "constant book leverage policy" as in He and Milbradt (2016), so the firm may isolate any other direct dilution motives to change future book leverage and bonds' stated maturities (Brunnermeier and Oehmke, 2013). This assumption also enables us to eliminate the influence of the flexibility in adjusting firm leverage (Childs et al., 2005; DeMarzo and He, 2021) and concentrate solely on the impact of timing flexibility provided by call provisions. Therefore, any retired bond is refinanced by issuing a new identical bond. Specifically, a callable bond can be refinanced early using an otherwise identical bond after the expiration of the retired callable's call protection period.[14] A non-callable bond will be refinanced using an otherwise identical bond only on its stated maturity date. With this debt flotation setting, we use m in Equations (2) and (3) to represent the frequency at which the firm rolls over the SB^c and SB^s. A larger (smaller) m given T implies a shorter (longer) maturity of the non-callable and hence a higher (lower) rollover frequency.

After bonds are issued, a firm will choose the time to default that best serves shareholders' interests. We follow He and Xiong (2012b) and He and Milbradt (2014) and assume that shareholders would bear gains and losses from refinancing activities. Specifically, if funds raised from issuing a new bond exceed (fall short of) the amount needed to repay the retired bond, then the resulting profit (deficit) will be allocated to (be covered by) shareholders as dividends (through additional equity issuance). The firm will

[14] Theoretical frameworks, such as Mauer (1993), Sarkar (2001), and Sarkar and Hong (2004), assume that a perpetual callable bond can be refinanced early with an otherwise identical non-callable bond. On the other hand, models such as Goldstein et al. (2001) and Chen (2010) allow for early refinancing of perpetual callable bonds with other perpetual callables. In the models of Chen et al. (2021) and Dangl and Zechner (2021), they further consider early refinancing activities by using non-perpetual callables. However, all of them ignore the presence of call protection periods when firms make their early refinancing decisions.

announce default whenever its debt repayment cannot be fulfilled via equity financing to maximize its shareholders' value (Chen, 2010). With this default policy, the CB^c issuer will also choose an earlier time to refinance its outstanding callable bond using another identical bond to maximize its shareholders' value.[15] Since the call protection length is stipulated *right before* the bond issuance, the debt refinancing call should only occur after the expiration of the period. In particular, the firm will choose the period length $P^* \in (0, T]$ to maximize the initial total levered firm value $V_0^{L.c}$ in Equation (2). The expected effective maturity of this T-year CB^c is defined as the expected value of the time to refinance the bond, $\kappa_C \in [P^*, T]$, under the risk-neutral probability measure.[16] Our numerical framework allows us to explore the refinancing decision κ_C depending on the predetermined call protection length under the settings of the constant book leverage policy and the lumpy debt maturity. Our framework also enables us to demonstrate how, in terms of interest costs, refinancing with callable bonds is more cost-effective for high-leverage firms compared to using short-term non-callable bonds.

2.3 Numerical Implementation

2.3.1 Evaluating Equity and Bonds

To the best of our knowledge, numerous existing models, such as Chen et al. (2021) and Dangl and Zechner (2021), account for a callable debt; however, they overlook the call protection period, rendering the timing of early refinancing independent of the period's duration. Since equity and bonds are viewed as contingent claims on their issuer's assets, we approach this issue by employing the forest structure pioneered in Liu et al. (2016) to simulate the asset value process of Equation (1) and feature lumpy debt maturity, as

[15]In the models proposed by Jarrow et al. (2010) and Ma et al. (2023), debt default is determined exogenously through a one-time Poisson process. However, this approach does not capture the interaction between call and default decisions, which has been highlighted by Acharya and Carpenter (2002) and Kim and Stock (2014).

[16]In other words, the optimal call protection length P^* is chosen to align the refinancing decision κ_C with the policy of maximizing the initial total levered firm value. Fischer et al. (1989) refers to this refinancing decision governed by predetermined call provision terms as the first best decision.

in Figure 2. We further denote the SB^c in Equation (2) (SB^s in Equation (3)) by $SB^c_{d_m}$ ($SB^s_{d_m}$), where the subscript represents that the non-callable bond's stated maturity date is $t = d_m$. In addition, we denote the CB^c by $CB^c_{d_c,\,d_n}$ in which the two subscripts d_c and d_n represent the callable bond's first call and stated maturity dates, respectively. Moreover, we denote the E^c (E^s) by $E^c_{d_m,\,d_n}$ ($E^s_{d_m,\,d_n}$) to indicate the levered equity when the firm employs the debt structure with two bonds due on dates d_m and d_n, respectively.

A degenerative version of the forest, a CRR binomial tree (Cox et al., 1979) with a length of time step $\Delta t = T/6$ in Panel A, can discretely simulate the firm value evolution of Equation (1) using the upper (downward) branch probabilities P_u (P_d) over time steps. The equity and non-callable bond values on the stated bond maturity date depend on the firm values for the terminal nodes of the tree. Their values at the root node can be found using backward induction in the tree. To tackle the rollover of non-callable bonds, we extend the tree in Panel A into the one in Panel B. Moreover, a typical forest composed of several CRR trees in layers are displayed in Panel C; each tree reflects the firm value evolution over a debt structure including a T-year callable bond with one permissible call date. We exploit the transition between two adjacent trees in the forest to model uncertain early refinancing of outstanding callable bonds. For example, the first (second) layer black (red) tree reflects the debt structure including the T-year $CB^c_{T/2,\,T}$ ($CB^c_{T,\,3T/2}$) that can be called at year $T/2$ (T) and matures at $t = T$ ($3T/2$). The transition from the black tree to the red one signifies the scenario in which the $CB^c_{T/2,\,T}$ is refinanced early through the proceeds from raising the $CB^c_{T,\,3T/2}$. Two adjacent tree structures that overlapped in the same time period simulate the values of all contingent claims based on whether or not an early refinancing is conducted. This approach enables us to pinpoint the best refinancing policy for a specific call date, such as node I at $t = T/2$ in the first layer tree. The extension to include multiple call dates during the call period will be detailed in Appendix A.1.2.

Bonds with Lumpy Maturities

To sketch our numerical implementation, we start by considering the debt structure described in Equation (3). For ease of illustration, we let $m = 2$ and $P = T/2$. That is, the debt structure contains two non-callable bonds; C_S and C_L (F_S and F_L) are the coupon rates (face values) of the two bonds, which have the same stated maturity of $T/2$ years. The subscript of SB^s denotes that the non-callable bonds will mature on the date of $t = T/2$, and the bond issuer does not roll over the two bonds on their maturity dates, as illustrated in Panel A. Thus, the two $SB^s_{T/2}$ and equity $E^s_{T/2,T/2}$ can be evaluated by simply using a standard backward induction procedure as follows. Under the setting of lumpy debt maturity, $SB^s_{T/2}$ holders (issuer) will receive (should repay) a coupon (after-tax coupon) plus face value on the maturity date of $t = T/2$ and will receive (should repay) a coupon (after-tax coupon) before the maturity date. Once these contractually-obligated payments cannot be fulfilled, the issuer announces default. After deducting liquidation expenses at a ratio of ω, the firm's residual assets are directed to $SB^s_{T/2}$ holders. We take, for example, the terminal node D at $t = T/2$. If the firm's asset value at node D is denoted by $\upsilon(D)$, then the levered equity value is expressed as:

$$E^s_{T/2,T/2}(\upsilon(D), T/2) = max\left(\upsilon(D) + \delta_{T/2} - (1-\tau)(C_S F_S + C_L F_L)\Delta t - (F_S + F_L)\,,\, 0\right), \quad (4)$$

in which the δ_t represents the cash payout for debt repayments and dividends. We follow Broadie and Kaya (2007) and set $\delta_t = V_t e^{q\Delta t} - V_t$; $V_{T/2} = \upsilon(D)$ in this example. The bond value is then expressed as:

$$SB^s_{T/2}(\upsilon(D), T/2\,|\,T/2) = \begin{cases} F_{\mathbb{M}} + C_{\mathbb{M}} F_{\mathbb{M}} \Delta t & \text{if } E^s_{T/2,T/2}(\upsilon(D), T/2) > 0, \\ (1-\omega)(\upsilon(D) + \delta_{T/2}) \times \alpha_{\mathbb{M}} & \text{otherwise,} \end{cases} \quad (5)$$

in which the subscript $\mathbb{M}$ can be S or L; the $\alpha_{\mathbb{M}}$ equals $F_{\mathbb{M}}/(F_S + F_L)$ since the two non-callable bonds are equally senior during the liquidation process.

Once equity and bond values for terminal nodes of the tree are determined, their values for all intermediate nodes can be calculated using the backward induction procedure as follows. The equity value at time t is evaluated using the following equation:

$$E^s_{T/2,T/2}(V_t, t) = \max\left(\underbrace{\delta_t - (1-\tau)(C_S F_S + C_L F_L)\Delta t}_{\substack{\text{U: cash dividends or}\\ \text{new equity issuance}}} + \underbrace{E^s_{T/2,T/2}(V_{t^+}, t^+)}_{\text{X1}},\, 0 \right). \tag{6}$$

In this equation, the term X1 represents the expected present equity value right after time t. In addition, a positive value of U indicates cash dividends distributed to shareholders as the cash payout δ_t surpasses the after-tax coupon payment. Conversely, a negative value represents a shortfall in the after-tax coupon payment, which necessitates new equity issuance. If the term X1 equity value cannot fulfill the shortfall (i,e., U+X1 < 0), then the firm announces default, and the shareholders receive nothing, as in Chen (2010). With this default policy, the bond value is:

$$SB^s_{T/2}(V_t, t \,|\, T/2) = \begin{cases} C_{\mathbb{M}} F_{\mathbb{M}} \Delta t + \underbrace{SB^s_{T/2}(V_{t^+}, t^+ \,|\, T/2)}_{\text{X2}} & \text{if } E^s_{T/2,T/2}(V_t, t) > 0, \\ (1-\omega)(V_t + \delta_t) \times \alpha_{\mathbb{M}} & \text{otherwise,} \end{cases} \tag{7}$$

in which the term X2 refers to the expected present value of future cash flows received by the bondholder. We take, for example, the equity value $E^s_{T/2,T/2}(\upsilon(A), T/3)$ and the bond value $SB^s_{T/2}(\upsilon(A), T/3 \,|\, T/2)$ for node A at $t = T/3$. They can be calculated using Equations (6) and (7) with the terms X1 and X2 being evaluated by the standard backward induction as:

$$e^{-r\Delta t}\left(P_u \times E^s_{T/2,T/2}(\upsilon(B), T/2) + P_d \times E^s_{T/2,T/2}(\upsilon(D), T/2) \right), \tag{8}$$

$$e^{-r\Delta t}\left(P_u \times SB^s_{T/2}(\upsilon(B), T/2 \,|\, T/2) + P_d \times SB^s_{T/2}(\upsilon(D), T/2 \,|\, T/2) \right), \tag{9}$$

respectively. Finally, the equity and bond values for the root node at $t = 0$ can be computed by mimicking the expression in Equations (8) and (9).

Rollover of Non-Callable Bonds

In Panel B, we model an otherwise identical firm adhering to the constant book leverage policy by repeatedly rolling over the two $T/2$-year non-callable bonds until we reach $t = 2T$ in an extended tree framework. Specifically, the two $SB^s_{T/2}$ will be repaid at $t = T/2$ through the proceeds from raising two otherwise identical SB^s_T that expire at $t = T$, and so on until $t = 2T$. Since the retired and newly-issued bonds are otherwise identical, shareholders would bear rollover gains or losses if funds raised from issuing a new bond exceed or fall short of the amount needed to repay the retired bonds. A firm will thus announce default if rollover losses exceed the equity value plus the cash payout. To evaluate the $E^s_{2T,2T}$ and the two $SB^s_{T/2}$ at $t = 0$ using a $2T$-year time span, we extend the $T/2$-year tree in Panel A into the $2T$-year one in Panel B, so we may consider the debt rollover on dates of $t = T/2$, T, and $3T/2$. The equity and bond values for the terminal nodes of this $2T$-year tree can be computed as those of the tree in Panel A. We take, for example, the terminal node H at $t = 2T$. The equity and bond values, $E^s_{2T,2T}(\upsilon(H), 2T)$ and $SB^s_{2T}(\upsilon(H), 2T \mid T/2)$, can be calculated by mimicking the expression in Equations (4) and (5). Their values on the SB^s_{2T} issue date, such as those for node G at $t = 3T/2$, can be determined using backward induction in the orange tree in Panel B, similar to the procedure used to find the values of $E^s_{T/2,\,T/2}$ and $SB^s_{T/2}$ for the root node in Panel A.

The impact of debt rollover on equity and bond values is illustrated as follows. We use node E at $t = T/2$ as an example and compare it with the scenario in Equation (4) where there is no rollover. The evaluation procedure at nodes F and G can follow this example. The equity value considering rollover is expressed as:

$$
\begin{aligned}
&E^s_{T/2,T/2}(\upsilon(E),T/2)\\
&= max\Bigg(\underbrace{E^s_{T,T}(\upsilon(E),T)}_{\substack{\text{Y1: levered equity value when}\\ \text{the two } SB^s_T \text{ are outstanding}}} \quad + \delta_{T/2} \underbrace{-(1-\tau)(C_S F_S + C_L F_L)\Delta t}_{\substack{\text{after-tax coupon payments}\\ \text{to the two } SB^s_{T/2}}} \\
&\qquad \overbrace{\underbrace{-(F_S+F_L)}_{\substack{\text{repayment of the}\\ \text{two maturing } SB^s_{T/2}}} \ \underbrace{+(1-\gamma)\Big(SB^s_T(\upsilon(E),T\,|\,T/2) + SB^s_T(\upsilon(E),T\,|\,T/2)\Big)}_{\text{Y2: proceeds from raising the two new } SB^s_T}}^{\text{Y: rollover gain and loss}}, \underbrace{0}_{\text{default}} \Bigg).
\end{aligned} \tag{10}
$$

The red terms represent values of the equity and newly-issued bonds that can be determined using backward induction in the red tree. If the firm does not engage in rollover, then term Y2 can be omitted, and term Y1 simplifies to $\upsilon(E)$, thereby reducing Equation (10) to Equation (4). The term Y can be positive or negative to reflect the rollover gain or loss born by shareholders. Since the two newly-issued SB^s_T and the retired $SB^s_{T/2}$ are otherwise identical, a lower $\upsilon(E)$ reveals a relatively poor firm's fundamental, which leads to smaller values of newly-issued bonds and a greater rollover loss (i.e., smaller term Y) that collectively reduces equity value and precipitates debt default. The value of the maturing $SB^s_{T/2}$ at node E can then be computed by mimicking the expression in Equation (5). By working backward in the $2T$-year tree until $t = 0$, we can find the initial values of $E^s_{T/2,T/2}$ and the two $SB^s_{T/2}$ for the root node if we assume the three debt rollover cycles depicted in Panel B.

Refinancing with Callable Bonds

In Panel C, we consider a firm with the debt structure described in Equation (2). Also, we let $m = 2$ and $P = T/2$; this firm is thus identical to the one in Panel B with a single exception of the debt structure. Again, the $T/2$-year non-callable bond $SB^c_{T/2}$ and

T-year callable bond $CB^c_{T/2,\,T}$ will be evaluated using a $2T$-year time span. For ease of illustration, the firm is only allowed to either call and refinance $CB^c_{T/2,T}$ early at $t = T/2$ or just roll over the callable at $t = T$ with an otherwise identical bond, and so on until $t = 2T$. We exploit backward induction in the forest in Panel C to handle the evaluation.[17] Since T-year callable bonds are evaluated using a $2T$-year time span, the maturing $SB^c_{3T/2}$ and $CB^c_{T,3T/2}$ at $t = 3T/2$ in the second layer red tree will be repaid through the proceeds from raising two $T/2$-year SB^s_{2T}, which are identical to those in Panel B. Moreover, the $SB^c_{3T/2}$ and $CB^c_{3T/2,\,2T}$ at $t = 3T/2$ in the third layer green tree will also be refinanced with the two SB^s_{2T}.[18] The values of the two SB^s_{2T} on their issue date of $t = 3T/2$ can be computed using backward induction in the fourth layer orange tree, which is identical to the orange one in Panel B.

Next, we focus on the terminal nodes at $t = T$ in the first layer black tree, so we may evaluate equity $E^c_{T,\,T}$ and the two maturing bonds, SB^c_T and $CB^c_{T/2,\,T}$. We take node K for example. Since the firm will the SB^c_T and $CB^c_{T/2,\,T}$ through proceeds from raising otherwise identical $SB^c_{3T/2}$ and $CB^c_{3T/2,\,2T}$, the equity value is expressed as follows

[17]If the call date at $t = T/2$ is removed, T-year $CB^c_{T/2,T}$ is reduced to a T-year non-callable. We can also use this forest structure to evaluate the contingent claims within a capital structure comprising two non-callable bonds with different stated maturities. This method is employed for pricing in Panel B when m and P are set to other values (see Appendix A.1.2). For brevity's sake, we only go through the callable bond case here.

[18]Since T-year callable bonds are evaluated using a finite time span equal to $N \times T$ years, we note that these bonds cannot always be refinanced (early) with otherwise identical ones. In this illustrative example, it will occur at $t = 3T/2$ when $N = 2$. However, the impact of this ad hoc setting on the initial values of $SB^c_{T/2}$ and $CB^c_{T/2,\,T}$ is trivial when N is great enough. Our robustness check in Appendix A.1.3 confirms this argument.

to reflect the rollover:

$$
\begin{aligned}
&E^c_{T,T}(\upsilon(K),T)\\
&= max\Bigg(\underbrace{E^c_{3T/2,2T}(\upsilon(L),T)}_{\substack{\text{levered equity value when}\\ SB^c_{3T/2} \text{ and } CB^c_{3T/2,2T}\\ \text{are outstanding}}} + \delta_T \underbrace{-(1-\tau)(C_S F_S + C_L F_L)\Delta t}_{\substack{\text{after-tax coupon payments}\\ \text{to } SB^c_T \text{ and } CB^c_{T/2,T}}}\\
&\qquad \underbrace{-(F_S+F_L)}_{\substack{\text{repayment of}\\ \text{the maturing}\\ SB^c_T \text{ and } CB^c_{T/2,T}}} \underbrace{+(1-\gamma)\Big(SB^c_{3T/2}(\upsilon(L),T\,|\,T/2) + CB^c_{3T/2,2T}(\upsilon(L),T\,|\,T/2,T)\Big)}_{\substack{\text{proceeds from raising the new}\\ SB^c_{3T/2} \text{ and } CB^c_{3T/2,2T}}}, \underbrace{0}_{\text{default}} \Bigg).
\end{aligned}
\tag{11}
$$

This equation is similar to Equation (10). The green terms represent the values of the equity and newly-issued bonds that can be determined using backward induction in the third layer green tree. This backward induction procedure enables these green terms at node L to account for refinancing events occurring at $t = 3T/2$; such an event could be either the one in which the maturing $SB^c_{3T/2}$ is repaid alone by issuing the SB^c_{2T} (i.e., proceed to the light green tree), or the one in which both the $SB^c_{3T/2}$ and $CB^c_{3T/2,\,2T}$ are refinanced with the two SB^s_{2T} (i.e., transfer to the orange tree). The values of the maturing $CB^c_{T/2,\,T}$ and SB^c_T at node K are then expressed based on the equity value as:

$$
CB^c_{T/2,T}(\upsilon(K),T\,|\,T/2,T) = \begin{cases} F_L + C_L F_L \Delta t & \text{if } E^c_{T,T}(\upsilon(K),T) > 0,\\ (1-\omega)(\upsilon(K)+\delta_T)\times\alpha_L & \text{otherwise;} \end{cases}
$$

$$
SB^c_T(\upsilon(K),T\,|\,T/2) = \begin{cases} F_S + C_S F_S \Delta t & \text{if } E^c_{T,T}(\upsilon(K),T) > 0,\\ (1-\omega)(\upsilon(K)+\delta_T)\times\alpha_S & \text{otherwise,} \end{cases}
\tag{12}
$$

in which α_L and α_S equals $F_L/(F_S+F_L)$ and $F_S/(F_S+F_L)$, respectively, due to their equal seniority during the liquidation process.

Working backward from the terminal nodes of the first black layer tree at $t = T$ to the intermediate nodes at $t = T/2$, we now evaluate equity and bonds based on whether or not $CB^c_{T/2,T}$ is refinanced early. We begin with node I: according to the fundamental $\upsilon(I)$, the firm will either choose to refinance $CB^c_{T/2,T}$ early and roll over $SB^c_{T/2}$ simultaneously, roll over $SB^c_{T/2}$ alone, or declare default. Since the optimal decision serves shareholders' best interests, the equity value is expressed as the maximal values of these three choices as follows:

$$
\begin{aligned}
&E^c_{T/2,T}(\upsilon(I), T/2) \\
&= max\Bigg(\underbrace{\color{red}{E^c_{T,3T/2}(\upsilon(J), T/2)}}_{\substack{\text{levered equity value when} \\ SB^c_T \text{ and } CB^c_{T,3T/2} \\ \text{are outstanding}}} + \delta_{T/2} - (1-\tau)(C_S F_S + C_L F_L)\Delta t \\
&\qquad \overbrace{\underbrace{-(F_S + K_{T/2})}_{\substack{\text{repayment of} \\ SB^c_{T/2} \text{ and } CB^c_{T/2,T}}} \underbrace{+(1-\gamma)\Big(\color{red}{SB^c_T(\upsilon(J), T/2\,|\,T/2)} + \color{red}{CB^c_{T,3T/2}(\upsilon(J), T/2\,|\,T/2, T)}\Big)}_{\substack{\text{proceeds from raising the new} \\ SB^c_T \text{ and } CB^c_{T,3T/2}}}}^{\text{Z1: gain or loss from rollover and early refinancing}}, \\
&\qquad \underbrace{\color{gray}{E^c_{T,T}(\upsilon(I), T/2)}}_{\substack{\text{levered equity value when} \\ SB^c_T \text{ and } CB^c_{T/2,T} \\ \text{are outstanding}}} + \delta_{T/2} - (1-\tau)(C_S F_S + C_L F_L)\Delta t) \\
&\qquad \overbrace{\underbrace{-F_S}_{\substack{\text{repayment of} \\ \text{the maturing} \\ SB^c_{T/2}}} \underbrace{+(1-\gamma)\color{gray}{SB^c_T(\upsilon(I), T/2\,|\,T/2)}}_{\substack{\text{proceeds from raising} \\ \text{the new } SB^c_T}}}^{\text{Z2: rollover gain or loss}}, \underbrace{0}_{\text{default}} \Bigg).
\end{aligned}
\tag{13}
$$

The first piece of equity value refers to the one when early refinancing is conducted; $CB^c_{T/2,T}$ is called back at the call price $K_{T/2}$ plus accrued interest.[19] The three red terms represent the values of the equity and newly-issued bonds that can be determined using backward induction in the second layer red tree. The second piece refers to the equity

[19] $\upsilon(I)$ is equal to $\upsilon(J)$ since the firm value evolution is a lognormal diffusion process, although the two tree nodes correspond to two different debt structures. The same holds for $\upsilon(K)$ and $\upsilon(L)$ in Equation (11).

value when the firm chooses to keep $CB^c_{T/2,T}$; $SB^c_{T/2}$ is repaid alone through proceeds from raising SB^c_T. The two gray terms represent the values of equity and the newly-issued bond that can be, on the other hand, determined using backward induction in the first layer gray tree. The choice between the first and second pieces illustrates the firm's flexibility in choosing the optimal time to refinance $CB^c_{T/2,T}$, thus minimizing losses (or maximizing gains) as represented by terms Z1 and Z2 from these activities. Compared to the equity value described by Equation (10) in Panel B, which refers to the rollover scenario, this timing flexibility enhances equity value and delays a default announcement. Consequently, it helps the firm that seeks to minimize bankruptcy costs related to refinancing activities, thus increasing the overall levered firm values.

Assuming the link between endogenous default and early refinancing action as described earlier, the value of $CB^c_{T/2,T}$ is determined based on equity value as:

$$CB^c_{T/2,T}(\upsilon(I), T/2\,|\,T/2, T) = \begin{cases} C_L F_L \Delta t + K_{T/2} & \text{if } E^c_{T/2,T}(\upsilon(I), T/2) > 0 \\ & \text{and call is announced,} \\ C_L F_L \Delta t + \underbrace{CB^c_{T/2,T}(\upsilon(I), T^+/2\,|\,T/2, T)}_{\text{X3}} & \text{if } E^c_{T/2,T}(\upsilon(I), T/2) > 0 \\ & \text{and call is not announced,} \\ (1-\omega)(\upsilon(I) + \delta_{T/2}) \times \alpha_L & \text{otherwise.} \end{cases}$$

Here, the term X3 refers to the expected present bond value right after $t = T/2$, and can be determined using backward induction in the first layer gray tree. In addition, since $SB^c_{T/2}$ is maturing at node I, it can be evaluated by mimicking the expression in Equation (12). Finally, by working backward again from the intermediate nodes at $t = T/2$ to the root node in the first layer tree, we can find the initial values of $E^c_{T/2,T}$, $SB^c_{T/2}$, and $CB^c_{T/2,T}$, considering all related refinancing activities depicted in Panel C.

Our above backward induction procedure will be detailed in Appendix A.1.1. We can achieve at least three extensions from the forest in Panel C without difficulty. The first one is from $m = 2$ into $m > 2$ to shorten the maturity of SB^c and thus increase rollover frequency. The second one is from using a $2T$-year time span into using an NT-year time span, in which N is sufficiently large and thus able to approximate the infinite time horizon adopted by most structural credit risk models. The third one is from a single call date into multiple call dates during the stipulated call period. These extensions will be detailed in Appendix A.1.2. The robustness of our numerical implementation when we use this tree-based method is confirmed by our quantitative results in Appendix A.1.3.

2.3.2 Estimating the Expected Effective Maturity for a Callable Bond

As we discussed earlier, the expected effective maturity of a callable bond is defined as the expected duration until the bond is refinanced under the risk-neutral probability measure. We estimate the expected effective maturity for the T-year callable bond $CB^c_{T/2,T}$ using backward induction in the black CRR tree of the forest in Panel C. For ease of illustration, we let $EEMat(V_t, t)$ denote the expected remaining time to refinance the callable at time t, given the firm's asset value is V_t. Our goal is to determine $EEMat(V_0, 0)$, which is the expected remaining time to refinance the callable at the root node of the forest. We let $CProb(V_t, t)$ denote the conditional probability that the callable will be refinanced at or after time t. Starting at the terminal nodes of the black tree at $t = T$, we set $CProb(V_T, T) = 1$ and $EEMat(V_T, T) = 0$ if the firm rolls over $CB^c_{T/2,T}$ on its stated bond maturity T like node K. Conversely, if the firm announces default, then both $CProb(V_T, T)$ and $EEMat(V_T, T)$ are set to 0.

When we work backward from the terminal nodes, the expected remaining time to refinance the callable at any intermediate node is expressed as:

$$EEMat(V_t, t) = \begin{cases} 0 & \text{if the bond is defaulted or refinanced,} \\ \Delta t + \underbrace{NextEEMat(V_{t+\Delta t},\, t + \Delta t)}_{\text{X4}} & \text{if the bond is outstanding.} \end{cases} \tag{14}$$

Here, Δt is the length of a time step for the CRR tree; for example, $\Delta t = T/6$ in Figure 2.

The term X4 indicates the expected value of the $EEMat(V_{t+\Delta t}, t+\Delta t)$, which is equal to:

$$\frac{P_u \times CProb(V_t u, t+\Delta t)}{CProb(V_t, t)} EEMat(V_t u, t+\Delta t) + \frac{P_d \times CProb(V_t d, t+\Delta t)}{CProb(V_t, t)} EEMat(V_t d,\, t+\Delta t).$$

P_u and P_d represent the branching probabilities in the CRR tree, while the parameters u and d determine the potential states of the firm's asset value. Specifically, from an initial value V_t at time t, the firm's asset value may either increase to $V_t u$ or decrease to $V_t d$ in the next time step. Given $EEMat(V_t, t) > 0$ (i.e., the bond is outstanding at time t), the denominator term is defined as follows:

$$CProb(V_t, t) = P_u \times CProb(V_t u, t+\Delta t) + P_d \times CProb(V_t d, t+\Delta t).$$

If $EEMat(V_t u, t+\Delta t)$ or $EEMat(V_t d, t+\Delta t)$ equals 0 due to early refinancing at time $t+\Delta t$, then the corresponding numerator term $CProb(V_t u, t+\Delta t)$ or $CProb(V_t d, t+\Delta t)$ is set to 1, as in node I of Panel C. Conversely, if $EEMat(V_t u, t+\Delta t)$ or $EEMat(V_t d, t+\Delta t)$ equals 0 due to a default announcement, then $CProb(V_t u, t+\Delta t)$ or $CProb(V_t d, t+\Delta t)$ is set to 0. Finally, we can calculate the expected remaining time to refinance the callable bond at the root node by recursively applying Equation (14).

3 Data and Calibration

In this section, we first describe how we collect bond-level and firm-level raw data, and then introduce the estimation procedure for variables characterizing our data. Finally, we describe how our model parameters are calibrated, so we may capture the important features of the collected data.

3.1 Data

We use the Mergent Fixed Income Securities Database (Mergent FISD) for our bond-level data. To focus on bonds subject to default risk, we exclude bonds issued by government-sponsored entities. In addition, we only include bonds whose redemption effective dates lie between their offering dates and stated maturity dates.[20] If the bonds are callable, we only consider the ones whose first call dates are set between their offering and stated maturity dates. We supplement the data on first call dates in Mergent FISD with the data from Bloomberg and the Securities Data Company (SDC) Platinum, following our procedure in Appendix A.2.

To retrieve corresponding firm-level data, we match our collected bond issuers to the firms in Compustat.[21] We only consider firms having at least three consecutive annual records in Compustat and three consecutive annual observations of public bonds outstanding in Mergent FISD. Our final sample includes 5,148 U.S. firms and 80,743 firm-year observations during the period 1990–2018. The sample covers 121,978 bonds, including 41,670 callable and 80,308 non-callable bonds.

To clearly illustrate the data characteristics and the relation between bond-level and firm-level variables, we first denote the length of stated bond maturity (call protection) by *BondStaM* (*BondCProt*), whose value is the time span in years between a bond's offering date and stated maturity date (first call date). In addition, we denote the length of effective bond maturity by *BondEffM*, whose value is the time span in years between the bond's offering date and redemption effective date. The time span eliminated from the original bond's life due to early redemption, *BondElim*, can thus be defined as *BondStaM*−*BondEffM*. For the sake of comparison, two relative measures *BondCProtR* (hereafter, call protection ratio) and *BondElimR* (hereafter, elimination ratio) are further defined

[20] In our paper, we only consider those effective dates related to five action types as in Xu (2018): 1) call, 2) repurchase, 3) tender offer, 4) refunded, and 5) mature. For more details on action types, see Appendix A.2.

[21] The data matching procedure is detailed in Appendix A.3.

as *BondCProt*/*BondStaM* and *BondElim*/*BondStaM*. A smaller *BondCProtR* refers to a shorter call protection period. Similarly, a greater *BondElimR* implies an earlier bond redemption. Finally, we denote a bond coupon rate in percentages by *BondCoupon*.

In Table 2, we compare the sample characteristics of callable bonds with those of non-callables. The mean *BondStaM* of the callable bonds is longer than that of the non-callables,[22] which echoes argument of Robbins and Schatzberg (1986) that embedding call provisions in bonds is a useful substitute for stating shorter bond maturities. We note that the mean *BondEffM* of the callable bonds is 4.55 years, which is close to the mean *BondStaM* of the non-callables (4.58 years). Furthermore, the median *BondCProt* of the callable bonds is 2.98 years, which is also close to the median *BondStaM* of the non-callables (3.01 years). On the other hand, the median ordinal rating of the callable bonds is 8 (i.e., BBB+), while that of the non-callables is 5 (i.e., A+). This finding is consistent with the observation of Brown and Powers (2020) that callable bonds have, on average, poorer ratings than non-callable bonds. In addition, the mean number of restrictive covenants for callable bonds is greater than that for non-callables, which also echoes the finding of Billett et al. (2007) that covenant protection is increasing with bond maturity.

Some of our firm-level variables are defined based on bond-level data as follows. If the i-th firm had l bonds outstanding in year t, then the firm-level stated bond maturity in years (coupon rate in percentages) is defined as:

$$FirmStaM_{i,t} = \frac{1}{l}\sum_{j=1}^{l} BondStaM_{i,t,j}; \tag{15}$$

$$FirmCoupon_{i,t} = \frac{1}{l}\sum_{j=1}^{l} BondCoupon_{i,t,j}. \tag{16}$$

[22]This characteristic is consistent with observations in Crabbe and Helwege (1994), Chen et al. (2010), and Booth et al. (2014), but is inconsistent with those in Brown and Powers (2020). The key reason for this inconsistency is that Brown and Powers (2020) only consider the bond sample with the stated maturity of at least three years, and doing so may exclude many short-term non-callable bonds from their dataset.

We treat $FirmCoupon_{i,t}$ as the proxy for interest costs on outstanding corporate bonds in year t. Similarly, if i-th firm has l_c callable bonds outstanding in year t, $l_c \leq l$, firm-level call protection length is defined in ratio form as:

$$FirmCProtR_{i,t} = \frac{1}{l_c}\sum_{j=1}^{l_c} BondCProtR_{i,t,j}. \tag{17}$$

On the other hand, to measure how early a firm conducts bond redemption, we exploit the length of time span eliminated from the length of stated bond maturity. This firm-level measure *FirmElimR* is defined only when the premature redemption is conducted by the i-th firm in year t as follows:

$$FirmElimR_{i,t} = \frac{1}{l_e}\sum_{j=1}^{l_e} BondElimR_{i,t,j}, \tag{18}$$

in which $l_e \leq l$. Intuitively, a smaller $FirmCProtR_{i,t}$ refers to shorter call protection periods for l_c callable bonds outstanding in year t. Furthermore, a greater $FirmElimR_{i,t}$ means that l_e bonds are redeemed earlier.

Table 3 reports our firm characteristics summary. Our sample firms have a median *FirmStaM* of 10 years close to the median *BondStaM* of our collected callable bonds and the benchmark setting in several studies, including Chen et al. (2021) and Dangl and Zechner (2021). By observing the mean *FirmCProtR*, we find that, on average, the outstanding callable bonds of a firm spend nearly half of their life in the form of call protection. In Table 4, we further conduct univariate tests of differences in bond characteristics depending on two proxies for rollover risk: 1) the leverage level, *Leverage*, in Panel A, and 2) the level of debt in current liability (Cathcart et al., 2020), *Curlia*, in Panel B. We find that high-leverage firms' *BondStaM* is significantly longer than low-leverage firms', which corroborates the argument from Diamond (1991) and Childs et al. (2005) that higher-leverage firms tend to increase their bonds' stated maturities in order to reduce the risk of experiencing debt rollover frequently. We also notice that high-leverage and high-curlia firms have significantly smaller *BondEffM* and *BondCProt* (*BondCProtR*), as well as greater *BondElim* (*BondElimR*). This implies that these firms tend to issue callable bonds with shorter call protection periods and conduct bond redemption earlier. In particular, for 10-year callable bonds, those issued by high-leverage (high-curlia) firms feature call protection periods that are, on average, 1.3 years shorter than those issued by low-leverage (low-curlia) firms. Furthermore, high-leverage (high-curlia) firms tend to redeem these bonds approximately 1.1 (0.8) years earlier than their low-leverage (low-curlia) counterparts.

3.2 Calibration

To facilitate our quantitative analysis in the next section, we use the set of baseline parameters for market condition and firm characteristics that are consistent with those in the literature to calibrate standard structural credit risk models. The parameters for debt structure are calibrated to capture the important features of our collected data.

By following the estimates of Dangl and Zechner (2021), we begin by assuming that a firm's income is taxed at a constant statutory rate $\tau = 30.6\%$, which is calibrated to effective marginal tax rates as recorded in the Compustat MTR database. Second, since the median ratings of our collected bonds are investment-grade (i.e., BBB+ for callable bonds and A+ for non-callable bonds), we choose $\gamma = 0.5\%$ of the market value of a newly-issued bond, as in He and Xiong (2012b) for A-rated bonds. Third, according to the estimates in Huang et al. (2020), the average payout ratio for a sample of firms is 2.14%. In particular, the average for A-rated firms is 2.02% and for BB-rated firms is 2.15%. Since the average ordinal rating of our firm samples is close to 10 (i.e., BBB−), we choose $q = 2\%$ as in He and Xiong (2012b) due to the small variation in payout ratio across different ratings. Similarly, the estimates in Zhang et al. (2009) show that A-rated firms have an average firm value volatility of 21% and that BB-rated firms have an average of 23%. Due to the small variation in firm value volatility across different ratings, we choose $\sigma = 21\%$

since the average rating of our firm samples is close to the investment-grade. Fourth, the bankruptcy cost ω is referred to Glover (2016), who estimates the mean (median) firm's cost of default with 45% (37%) of the firm's asset value by applying a structural trade-off model of a firm with time-varying macroeconomic conditions. We adopt $\omega = 37\%$ as in Dangl and Zechner (2021).

Other model parameters are specified according to our collected data. The stated maturity of the callable bond T is set to 10 years, since the median *BondStaM* of our collected callables is close to 10 years, as displayed in Table 2. We then set the risk-free rate r to 4.61%, which is the median 10-year Treasury rate during 1990–2018 according to data from the Federal Reserve Board's H.15 Report. On the other hand, the parameter m in Equation (2) represents the frequency at which the firm rolls over the shorter-term T/m-year non-callable bond. Given T, a larger m results in shorter bond maturities, thereby increasing the frequency and risk of rollover, while a smaller m decreases them. We begin by setting $m = 10$, which establishes the non-callable bond's stated maturity at one year, following the baseline scenario used in He and Xiong (2012b). Since the non-callable is relatively short-term, we calibrate the ratio of its face value F_S to the total debt face value $F_S + F_L$ to the median *Curlia* of 8%, as shown in Table 3. For our sensitivity analysis, we then choose $m = 2$ to represent a low rollover-risk scenario under otherwise identical conditions. To estimate total debt face value, we first consider a firm's leverage as the ratio of the firm's asset value to equity value. We then calibrate the firm's leverage to the median *Leverage* of 2.61, as shown in Table 3. If the firm's current asset value is normalized to $V_0 = 100$ as in Leland and Toft (1996) and is treated as the sum of the total debt face value and equity value as in Eom et al. (2004), then the total debt face value is about 61.68, which coincides with the value chosen by He and Xiong (2012b).

4 Quantitative Analysis

In this section, we examine the quantitative implications of our framework that we proposed in Section 2 by using the parameters shown in Table 5. If not otherwise mentioned, bond coupon rates C_S and C_L are set such that bonds issued at $t = 0$ are priced at par. To facilitate our numerical implementation, the callable bond CB^c in Equation (2) is supposed to have discrete call dates; the embedded call provision enables the issuer to announce a call at par once per year after the call protection period expires.[23] In Section 4.1, we first investigate the optimal duration of call protection and how subsequent refinancing timing is influenced by this choice. We will estimate the expected effective maturity of the CB^c, which is defined as the expected remaining time to refinance the callable at $t = 0$, based on all possible refinancing timing. In Section 4.2, we study how sensitive the choice of call protection length and the expected effective maturity are to variant firm leverages and rollover frequencies. In Section 4.3, we explore the welfare implications of strategically using callable bonds by quantifying the difference in interest costs between refinancing with callable bonds and rolling over shorter-term non-callable bonds.

4.1 The Choice of Call Protection Length and Effective Maturity

We start by examining the choice of call protection length and the subsequent refinancing timing. *Ceteris paribus*, the price of a callable bond is lower when it has a shorter call protection period. This is because the issuing firm has more call dates available to redeem the callable early at predetermined call prices, which comes at the expense of bondholders.

[23]The setting of callable at par allows for a comparison between refinancing with callable bonds and rollover of shorter-term non-callable bonds. Redemption at par narrows the comparison to exclusively examine the nuances of debt refinancing, either solely on stated maturity dates without flexibility or throughout stated call periods with multiple call dates. Chen et al. (2021) and Dangl and Zechner (2021) also adopt this setting.

When a callable bond is redeemed early at the call price below its prevailing market price, the firm transfers the wealth from bondholders to shareholders. In the face of this call risk, bondholders require a higher bond yield (i.e., call risk premium) for compensation when the callable is issued at par. This requirement totally offsets the aforementioned benefits shareholders might accrue from a shorter call protection period. Consequently, the firm remains unaffected by the choice of call protection length.

If a firm must frequently access debt markets for refinancing, then specifying a shorter call protection period will provide the firm with additional flexibility to adjust refinancing timing during the call period. This timing flexibility prevents the need to issue new bonds during times of financial strain, thus enhancing the firm's creditworthiness. Although a shorter call protection period strengthens the credit enhancement effect and thus decreases bondholders' required default risk premiums, it simultaneously raises required call risk premiums. Therefore, to maximize the initial total levered firm value, the firm must balance the benefits of increased timing flexibility and the costs related to call risk when making the length arrangement.[24] This benefit-cost trade-off is illustrated by the hump-shaped curve in Panel A of Figure 3. It shows that assigning a 3-year call protection for a 10-year callable bond (denoted by the star O) is the optimal choice for the calibrated firm if refinancing with callable bonds is conducted repeatedly. This assignment is close to the median *BondCProt* of 2.98 years displayed in Table 2. If a shorter call protection period is assigned to extend the call period, then the marginal costs from the incremental call risk premiums will outweigh the marginal benefits from declining default risk premiums. However, if a longer call protection period is assigned to shrink the call period, then the marginal costs from the incremental default risk premiums will outweigh the marginal

[24]Note that the initial total levered firm value $V_0^{L \cdot c}$ in Equation (2) can be expressed as follows, according to Leland (1994): $V_0^{L \cdot c} = V_0 + TB(V_0, 0) - BC(V_0, 0)$, in which $TB(V_0, 0)$ represents the present value of tax deductions from coupon payments (i.e., tax benefits), and $BC(V_0, 0)$ is the present value of bankruptcy costs. By adjusting the call protection length, the firm modifies the required default and call risk premiums. These changes affect the present values of tax benefits and bankruptcy costs, leading the firm to select the optimal protection length to maximize its initial total levered value. In a capital market with frictions, this strategy does not necessarily coincide with one that minimizes the coupon rate of the 10-year CB^c.

benefit from declining call risk premiums.

By strategically determining call protection lengths, firms can avoid issuing new bonds during financial downturns, thus minimizing bankruptcy costs associated with rollover risks and enhancing overall firm value. This increased value motivates firms to initiate refinancing earlier within call periods. Through proactive debt repricing and refinancing, firms secure gains for their shareholders, mitigating the potential for risk-shifting and underinvestment behaviors. As a result, effective maturity dates of callable bonds move towards the beginning of call periods. Our calibrated framework well captures this refinancing call policy. If timing flexibility is denoted by the distance between the dotted and dashed lines in Panel B, the expected refinancing policy along with the flexibility is illustrated by the significantly narrower (wider) gap between the expected effective maturity and the call protection length (stated maturity). The two asymmetric gaps echo the observed pattern of the effective maturity over the past three decades, as shown in Panel A of Figure 1. Our baseline prediction for the expected effective maturity of a 10-year callable bond with a 3-year call protection is near 4.2 years (denoted by the star O'), which is also close to the median *BondEffM* of 4.15 years displayed in Table 2. As a result, instead of the stated maturity of a callable bond, the call-to-shorten strategy makes its call protection length a more accurate measure of its real lifespan.

Our results in Figure 3 yield the following prediction:

Prediction 1. *With the flexibility to adjust debt refinancing timing during a call period, firms tend to conduct early refinancing near the beginning of the predetermined call period.*[25]

[25]This prediction seeks to explain observations post-1990, since we construct our calibrated model by primarily using data collected from that year onwards.

4.2 The Choice of Call Protection Length and Effective Maturity Concerning Variant Rollover Risk

We next examine how choices of call protection length and effective maturity are affected by different levels of firms' exposure to rollover risks. Throughout our analysis, we use firm leverage (proxied by total debt face value $F_S + F_L$) and rollover frequency m as indicators of risk, as in Childs et al. (2005) and He and Xiong (2012b). That is, the exposure to rollover risk increases as the total debt face value (rollover frequency) increases, *ceteris paribus*. Panel A of Figure 4 plots the optimal lengths of call protection over total debt face values for two different levels of rollover frequency. We observe that a firm with very low leverage tends to issue non-callable bonds due to minimal exposure to rollover risk. The firm chooses to align protection length with the stated maturity, since the costs of using callable bonds are greater than the benefits of moderating rollover risk via the callables. As total debt face value increases from 35 to over 40, the firm with rollover frequency $m = 10$ issues callable bonds.[26] Moreover, since the benefits of assigning a shorter call protection period increase at a faster rate than costs when rollover risk becomes prominent, the firm tends to choose a shorter protection length as its leverage is higher. We observe that the optimal protection length decreases from 5 to 1 years as the total debt face value increases from 40 to 85.[27] This decreasing pattern is less pronounced for the firm with rollover frequency $m = 2$ due to mild rollover risk. These results not only echo differences in bond characteristics, *BondCProt* and *BondCProtR* in Table 4, but also confirm that the desire for precautionary management of rollover risk motivates

[26] Chen et al. (2010) also identify this fact, but their focus lies in choosing whether to issue a callable or non-callable bond and how this decision relates to a firm's investment risk. In their numerical analysis, they argue that a high-leverage firm tends to issue a callable bond because the embedded call provision provides the flexibility to early retire the callable through internal funds. Doing so enables the firm to avoid the risk of having to repay the bond during a weak financial state due to a bad investment outcome. Instead of rollover risk, their framework targets at debt repayment risk arising from investment uncertainty.

[27] Perhaps surprisingly, the quantitative analysis indicates that choosing 6-year to 9-year call protection periods is suboptimal for a 10-year callable bond. That is, as firms issue callable bonds, their arrangement for call period length is at least 50% of the entire stated bond maturity. This result is actually supported by the data, except in the cases for which the samples include bonds with make-whole call provisions. Figure 3 in Powers (2021) also shows that call protection length rarely exceeds half of a bond's life.

the choice for shorter call protection periods (longer call periods).

Panel B illustrates the expected effective maturity corresponding to each optimal length of call protection length in Panel A. Due to timing flexibility, callable bonds are likely to be refinanced close to the start of predetermined call periods, as captured in *Prediction 1*. Consequently, as expected, there is a more pronounced disparity between expected effective maturity and stated maturity, especially when shorter call protection periods are incorporated. The disparity reflects the observed connection between high rollover risk and frequent debt refinancing (Ma et al., 2023).

Our results in Figure 4 yield the following prediction:

Prediction 2. *With higher exposure to rollover risk, firms tend to choose callable bonds with shorter call protection periods.*

Prediction 3. *The disparity between effective and stated maturity is more pronounced as firms choose shorter call protection periods.*

4.3 Welfare Implications of Strategically Using Callable Bonds

Finally, we examine the welfare implications of strategically using callable bonds by comparing the interest costs of refinancing with callable bonds to those of rolling over shorter-term non-callable bonds. Two otherwise identical firms are considered. The first firm's debt structure comprises a $10/m$-year SB^c and a 10-year CB^c with a P^*-year call protection, as expressed in Equation (2), and P^* is the optimal protection length under a given total debt face value. The other comprises a $10/m$-year SB^s and a P^*-year SB^s, as expressed in Equation (3). We align the stated maturity of the latter SB^s with the protection length P^* based on trends over the past two decades, as illustrated in Figure 1. We then pinpoint two goals. First, we aim to quantify the impact of stating a call period rather than just a single maturity date upon issuance by comparing the coupon rate of the CB^c with that of the P^*-year SB^s. Second, since using CB^c can enhance the firm's creditworthiness, we aim to examine how this effect influences the coupon rate of other bonds in the same debt structure. We illustrate this effect through the difference in coupon rates between the two otherwise identical $10/m$-year SB^c and SB^s.

Panel A (C) of Figure 5 displays differences in coupon rates between CB^c and the P^*-year SB^s over total debt face values. As anticipated, CB^c has a higher coupon rate than SB^s when the total debt face value is low. This difference primarily stems from the presence of the call risk premium and the higher default risk premium on a longer-term bond. With lower firm leverage, the mild rollover risk implies that granting timing flexibility cannot significantly reduce the default risk premium associated with the risk. However, this outcome changes when the total debt face value is high. By shortening the call protection period P^* to enhance timing flexibility (see Panel A of Figure 4), the firm can effectively moderate the increase in the default risk premium by suppressing the premium related to rollover risk. On the other hand, since P^* is smaller as the total debt face value is higher, the shorter SB^s magnifies the issuer's rollover risk, thereby leading to a surging coupon rate.[28] Although Robbins and Schatzberg (1986) argue that short-term non-callable and long-term callable bonds can act as perfect substitutes in managing conflicts between shareholders and bondholders, our results suggest that choosing callable bonds with short call protection might be a cost-effective solution for high-leverage firms.

Panel B (D) exhibits differences in coupon rates between the two otherwise identical $10/m$-year SB^c and SB^s. Since the use of CB^c can alleviate the risk of financial distress and thus help control overall interest costs, it is intuitive to observe that SB^c has a lower coupon rate than SB^s. By shortening the call protection length when the total debt face value is higher, the firm can effectively moderate the increase in coupon rates not just for CB^c itself but also for the shorter-term SB^c. Our findings thus suggest that providing

[28] Choosing a short-term bond would still help mitigate agency conflicts for high-leverage firms (Myers, 1977; Barclay et al., 2003), but the overall benefits of this choice are likely to be smaller due to the surging interest disbursement. To balance rollover risk, the literature typically recommends either reducing leverage (He and Xiong, 2012b; Dangl and Zechner, 2021) or lengthening stated maturities of non-callable bonds (Childs et al., 2005).

additional timing flexibility through the use of callable bonds also proves advantageous in managing rollover risk and controlling interest costs for firms that rely on short-term debt, such as financial firms (He and Xiong, 2012a; Della Seta et al., 2020) or small and medium-sized non-financial enterprises (Cathcart et al., 2020).

5 Empirical Evidence

Our quantitative analysis yields several predictions linking the choice of call protection length to a firm's exposure to rollover risk. Specifically, we find that firms tend to choose callable bonds with shorter call protection periods out of their stronger desire for precautionary management of rollover risk. Since callable bonds are likely to be refinanced near the start of predetermined call periods, the disparity between their effective and stated maturity is more pronounced as firms choose shorter call protection periods. Moreover, we investigate welfare implication of strategically using callable bonds by comparing interest costs of refinancing with callable bonds to those of rolling over shorter-term non-callable bonds. Our finding implies that using callable bonds can help control overall interest costs on outstanding corporate bonds. In this section, we discuss evidence for these predictions.

5.1 Methodology

To relate these predictions to evidence, we pay attention to real-world corporate debt refinancing activities. In particular, we target activities that we consider examples of "early" debt refinancing, based on the definition by Xu (2018) as follows:

1. The issue date of a newly-issued bond should fall within a 3-month window centered around the retirement date of a previously-issued bond.

2. The refinancing activity defined in 1) should occur at least 6 months before the stated maturity date of the retired bond.

In the first part of our definition, the timing of a debt refinancing activity is based on the issue date of the newly-issued bond rather than the retirement date of the previously-issued bond. Meanwhile, early refinancing activity defined in the second part of our definition further implies that retired bonds should be callable or redeemable, and that such bonds must be redeemed at least 3 months before their stated maturity dates. We identify retired bonds by using the following four methods of redemption that affect a bond's outstanding amount: 1) fixed-price calls, 2) make-whole calls, 3) repurchases, and 4) tender offers. Since firms can reshuffle bond contract terms at their discretion during debt refinancing processes, we follow Xu (2018) and treat these early refinancing activities as the firms' precautionary actions of debt structure management.

Our empirical analysis at the firm level proceeds as follows. We begin by defining a binary variable $D(EarlyRefinance)_{i,t}$ to mark early refinancing activities in year t for the i-th firm. If this firm issued a qualified new bond for debt refinancing in year t, then $D(EarlyRefinance)_{i,t}$ equals one; otherwise, it equals zero. To examine how early refinancing activities influence the characteristics of firms' outstanding (callable) bonds, we run panel regressions with the following specifications:

$$\begin{aligned} FirmCProtR_{i,t} &= \delta_0 + \delta_1 D(EarlyRefinance)_{i,t} + \boldsymbol{\delta_i'} \boldsymbol{Controls_{i,t}} + \varepsilon_{i,t}, &(19)\\ FirmElimR_{i,t} &= \gamma_0 + \gamma_1 D(EarlyRefinance)_{i,t} + \boldsymbol{\gamma_i'} \boldsymbol{Controls_{i,t}} + \epsilon_{i,t}, &(20)\\ FirmCoupon_{i,t} &= \phi_0 + \phi_1 D(EarlyRefinance)_{i,t} + \boldsymbol{\phi_i'} \boldsymbol{Controls_{i,t}} + e_{i,t}. &(21)\end{aligned}$$

These three dependent variables that characterize the i-th firm's outstanding bonds in year t are defined in Equations (17), (18), and (16) in Section 3.1, respectively. $FirmCProtR_{i,t}$ is the average call protection ratio, $FirmElimR_{i,t}$ is the average elimination ratio, and $FirmCoupon_{i,t}$ is the average coupon rate on outstanding corporate bonds. We provide an illustrative example in Table A.4 of Appendix A.4 to detail how the three variables are constructed based on bond-level variables over time. On the other hand, the column vector $\boldsymbol{Controls_{i,t}}$ represents the control variables related to firm characteristics and credit market condition, including firm size ($ln(Assets)$), leverage ratio ($Leverage$), market-to-book ratio ($M/B\ Ratio$), tangible assets ($Tangibility$), earnings before interest, taxes, depreciation, and amortization ($EBITDA$), cash and short-term investment ($Cash$), annual equity return ($Equity\ return$), and corporate term spread ($Termspread$). These definitions are also detailed in Appendix A.4. To control for time-unvarying unobservables that might also affect a given firm's choices of call protection length or debt refinancing timing, we include firm fixed effects. We also include year fixed effects to control for the interest rate and observable credit market conditions affecting the aforementioned firm's choices.

Using these three panel regressions, we aim to examine how rollover risk exposure influences firms' preference for shorter call protection periods, the divergence of effective maturities from stated maturities, and the interest costs after the strategic use of callable bonds. To address these targets, we run these three regressions separately for firms with high and low rollover risk exposure. We employ three measures of rollover risk. First, we adopt leverage ratio ($Leverage$) according to Childs et al. (2005) and He and Xiong (2012b). Second, we use the level of debt in current liability ($Curlia$) defined as the portion of debt in current liability (DLC in Compustat) to DLC plus long-term debt due in more than one year ($DLTT$ in Compustat), as in Duchin et al. (2010). Third, we employ debt refinancing intensity (RI) proposed by Friewald et al. (2022). We follow their procedures by setting missing values of long-term debt due within one year ($DD1$ in Compustat), in years two to five ($DD2$, $DD3$, $DD4$, and $DD5$ in Compustat), and $DLTT$ to zero. Following their approach, we also apply two additional filters: 1) we remove the observations whose total debt (i.e., $DD1 + DLTT$) is greater than total assets, and 2) we remove the observations whose $DLTT$ is lower than the sum of $DD2$, $DD3$, $DD4$, and $DD5$. To capture the high rollover-risk nature of financial firms (He and Xiong, 2012a; Della Seta et al., 2020), we reformulate our refinancing intensity measure as $RI = DD1/(DD1 + DLTT)$. Observations are classified as the high (low) rollover-risk group in year t as their *Leverage* is beyond (below) the median *Leverage* in the same year. Classification via *Curlia* or RI also proceeds in the same manner.

5.2 Results

In our first step, we examine the relationship between early refinancing activities and leverage dynamics. We calculate average leverage ratios before, during, and after the early refinancing year for firms conducting early refinancing activities, and then we make pairwise comparisons to discern patterns. Moreover, we extend this analysis by juxtaposing the results for firms that did not conduct early refinancing activities during the same fiscal year. We report in Table 6 the leverage dynamics across three distinct phases. Two findings are worth noting. First, early refinancing activities do not result in significant changes in leverage ratio, as it seems that firms do not considerably adjust their leverage ratios through the channel of early refinancing.[29] Second, firms conducting early refinancing exhibit higher leverage ratios than those that do not, which reveals a strong connection between firms with high leverage and their engagement in precautionary actions. The main driver for this connection is most likely the management of rollover risk, especially considering the decreased stated maturity of corporate debt in the U.S. over the past four decades (Custódio et al., 2013; Butler et al., 2022), as displayed in Figure 1.

Next, we examine how firms reshuffle call protection length for their outstanding callable bonds through early refinancing activities. In Table 7, we present the results of our regression analysis with *FirmCProtR* as the dependent variable. The key independent variable of interest is $D(EarlyRefinance)$. From the analysis conducted on the full sample in column (1), we can see that the coefficient estimate for this key independent variable is negative and statistically significant. This negative relation reveals that firms reduce their call protection ratios by conducting early refinancing in order to enhance

[29] However, this observation contrasts with settings adopted by several models, such as Leland (1998), Goldstein et al. (2001), Chen (2010), Chen et al. (2021), and Dangl and Zechner (2021).

timing flexibility offered by callable bonds. This finding prompts the question: what is the main driving force for this trend? The evidence presented in the following columns confirms *Prediction 2* from Section 4.2, which anticipates that firms with higher rollover risks tend to choose callable bonds with shorter call protection periods. In columns (2) and (3), we find that high-leverage firms significantly reduce their call protection ratios by 3.02% through early refinancing over time, compared to just a 0.95% reduction in the low-leverage counterparts. This prediction is also confirmed by our results for firms with high *Curlia* or *RI* shown in columns (4) and (6), respectively Although coefficient estimates for $D(EarlyRefinance)$ are also negative and statistically significant in columns (5) and (7) for firms with low *Curlia* and *RI*, the coefficient values are much higher than those in the former two columns.

We next focus on *Prediction 3* from Section 4.2, investigating whether firms tend to refinance their bonds sooner when opting for shorter call protection ratios, thus leading to a more pronounced divergence between effective and stated maturities. We present the results of our regression analysis with *FirmElimR* as the dependent variable in Table 8. From the analysis conducted on the full sample in column (1), we observe that the coefficient estimate for $D(EarlyRefinance)$ is positive and statistically significant; early refinancing activities notably deviate effective maturities from stated maturities. If this positive coefficient estimate is juxtaposed with the negative estimate for $D(EarlyRefinance)$ as in column (1) of Table 7, then the lower call protection ratio indeed leads to a more pronounced disparity between effective and stated maturity over time. Frequent debt refinancing and the selection of shorter call protection periods are highly correlated.

Similar dynamics also appear in our analysis of sub-sample groups. First, we note that all coefficient estimates for $D(EarlyRefinance)$ from columns (2) to (7) of Table 8 are positive and statistically significant. The overall pronounced deviations between effective and stated maturities confirm *Prediction 1* from Section 4.1, which anticipates that firms with timing flexibility tend to refinance their callable bonds near the start of predetermined call periods. However, the extent of these deviations varies depending on the chosen call protection ratios. Firms opting for smaller call protection ratios (i.e., firms with high *Leverage*, *Curlia*, and *RI* in columns (2), (4), and (6) of Table 7) tend to conduct refinancing earlier, which notably shortens effective maturities. Therefore, we observe greater coefficient estimates for $D(EarlyRefinance)$ in columns (2), (4), and (6) of Table 8 than for those in columns (3), (5), and (7). For example, in columns (4) and (5), firms with intense refinancing needs reduce effective maturity by about 18% over time, compared to just about a 7% reduction for firms with less frequent refinancing needs. These results align well with *Prediction 3*.

Finally, we examine firms' interest costs on their outstanding corporate bonds through our regression analysis with $FirmCoupon$ as the dependent variable in Table 9. Since column (1) of Table 7 exhibits a decreasing trend of the call protection ratio, we anticipate a positive coefficient estimate for $D(EarlyRefinance)$ in column (1) of Table 9 to reflect increased call risk premiums on callable bonds. However, this estimate is negative but not statistically significant. This inconsistency underscores the advantages of financial flexibility provided by callable bonds in enhancing firms' creditworthiness. While decreased call protection ratios raise call risk premiums on callable bonds, the resulting credit enhancement reduces default risk premiums on both callable and non-callable bonds. That, in turn, leads to either no significant change or even a slight reduction in the average coupon rate. Since the credit enhancement effect is expected to be stronger for firms with higher rollover risk, we observe a more pronounced reduction in columns (2), (4), and (6) than for those in columns (3), (5), and (7). For example, in column (6), our regression on the sample of firms with high *RI* reveals a negative and statistically significant coefficient for $D(EarlyRefinance)$ at the 10% level. However, in column (7), our regression on the sample of firms with low *RI* does not show statistical significance for the same variable.

6 Conclusion

Firms can make more flexible debt maturity decisions with call provisions, which allows firms to issue long-term bonds and then conduct early redemption, effectively shortening bonds' terms. This paper addresses how the gap between the effective and stated maturities of callable bonds reveals the importance of call protection periods, which have become a more accurate proxy for a bond's real lifespan. We introduce a novel theoretical framework to explore the choice of call protection length, focusing on early debt refinancing.

We use a framework that emphasizes timing flexibility enabled by call periods, bound by call protection expiration and stated maturity dates. This flexibility allows firms to refinance callable bonds at strategic times, so they may avoid new issuances during financial downturns and enhance their creditworthiness. While shorter call protection periods enhance credit profiles by lowering default risk premiums, they also increase call risk premiums. This trade-off leads us to predict that firms facing higher rollover risks, particularly those with high leverage or frequent short-term debt, are likely to opt for shorter call protection periods as a proactive risk management strategy.

Moreover, our framework connects debt refinancing timing to call protection length, showing that strategic flexibility can increase overall firm value by reducing bankruptcy costs. This increased value encourages earlier refinancing, helping firms maintain value gains for shareholders and mitigating risk-shifting or underinvestment behaviors. Thus, the call-to-shorten refinancing strategy pulls the effective maturity date toward the beginning of the call period, establishing call protection length as a crucial indicator of a bond's effective maturity.

Empirical evidence supports our findings, showing a declining trend in call protection ratios driven primarily by firms with high rollover risks that tend to refinance their callable bonds early. We also find a stable or slightly reduced average coupon rate despite shorter call protection ratios, underscoring the financial benefits of enhanced flexibility. Our study not only extends our understanding of the strategic use of callable bonds to manage rollover risk but also suggests that further research on various facets of call provisions could enrich our understanding of implications for the corporate bond markets.

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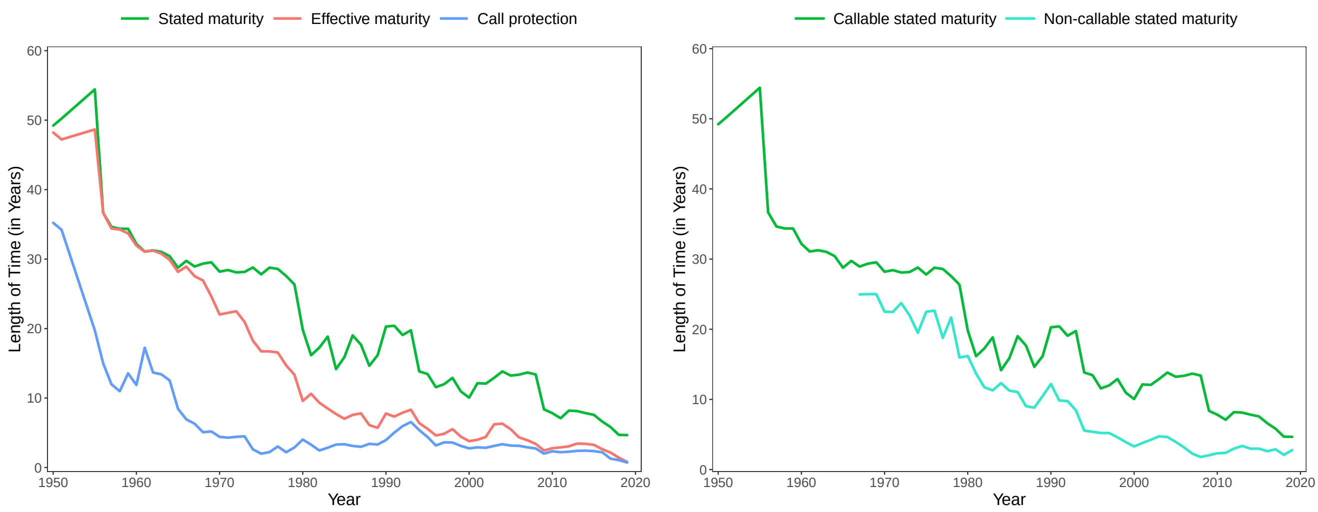
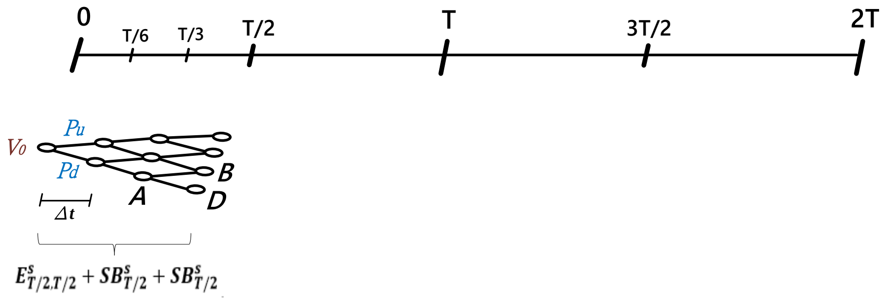
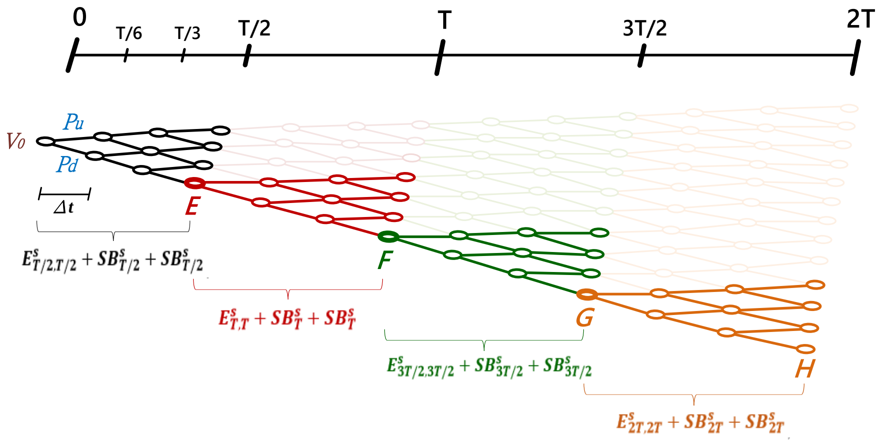


A. Callable bond B. Callable vs. non-callable bond

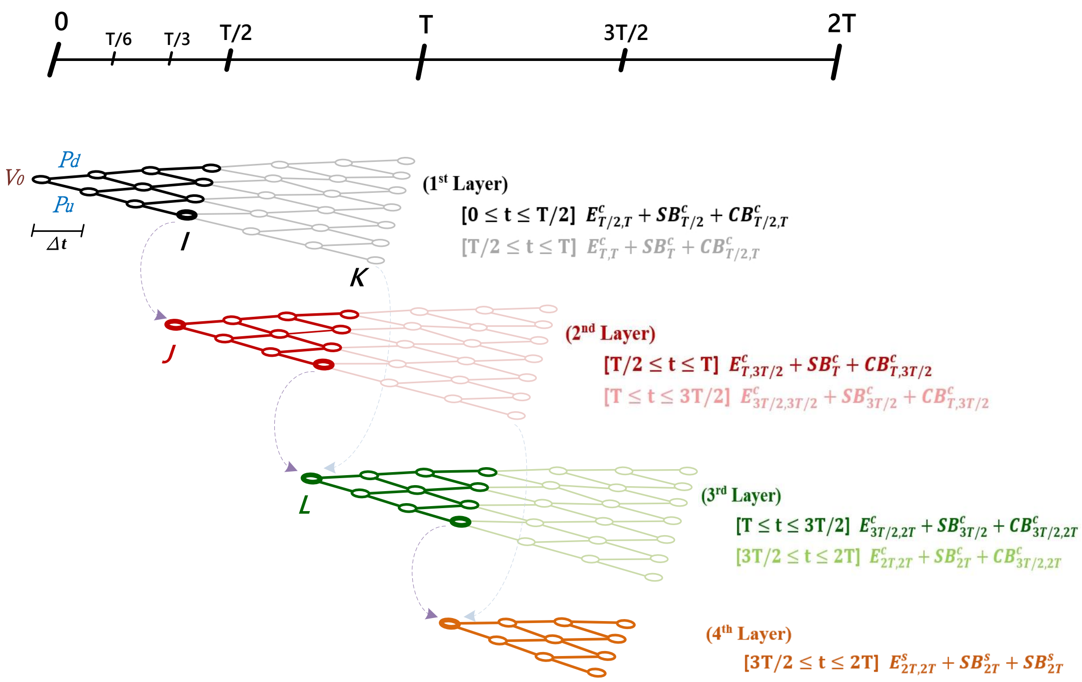
Figure 1: The average effective maturity (call protection length) for callable bonds and the average stated maturities for callable and non-callable bonds. We collect corporate bonds issued between 1950–2019 (denoted by the x-axis) from the Mergent Fixed Income Securities Database and exclude callable bonds that are still outstanding in December 2019. The green curves in both panels indicate the average stated maturity of callable bonds. The red and blue curves in Panel A indicate the average effective maturity and call protection length of callable bonds, respectively. The cyan curve in Panel B represents the average stated maturity of non-callable bonds. The stated maturity of a bond is the time span in years between its offering and maturity dates. The effective maturity of a callable bond is the time span between its offering and redemption effective dates. The call protection length of a callable bond is the time span between its offering and first call dates.



A. Expiration of a non-callable bond

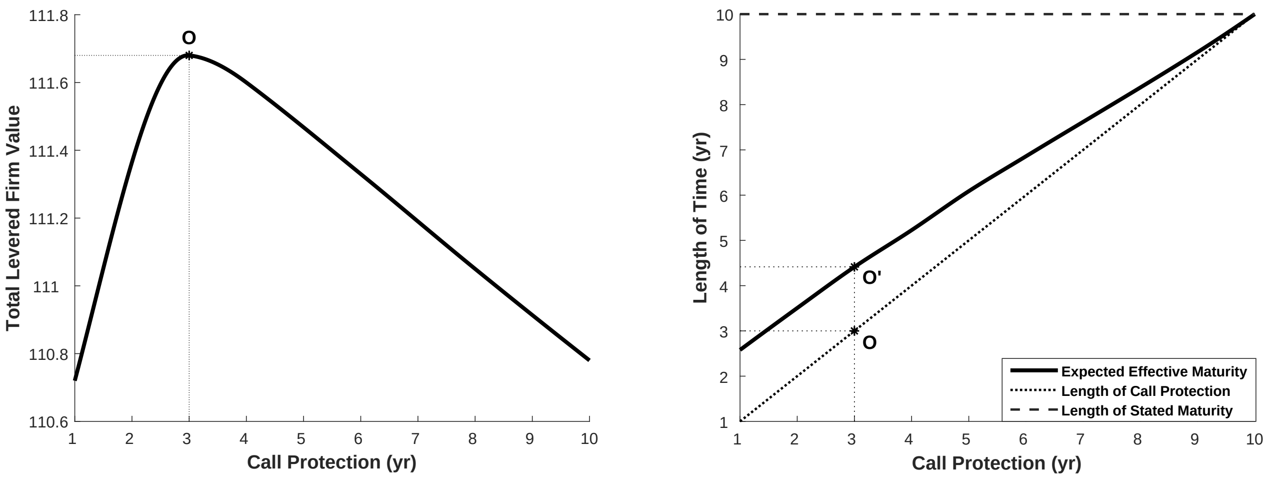


B. Rollover of non-callable bonds



C. Refinancing with callable bonds

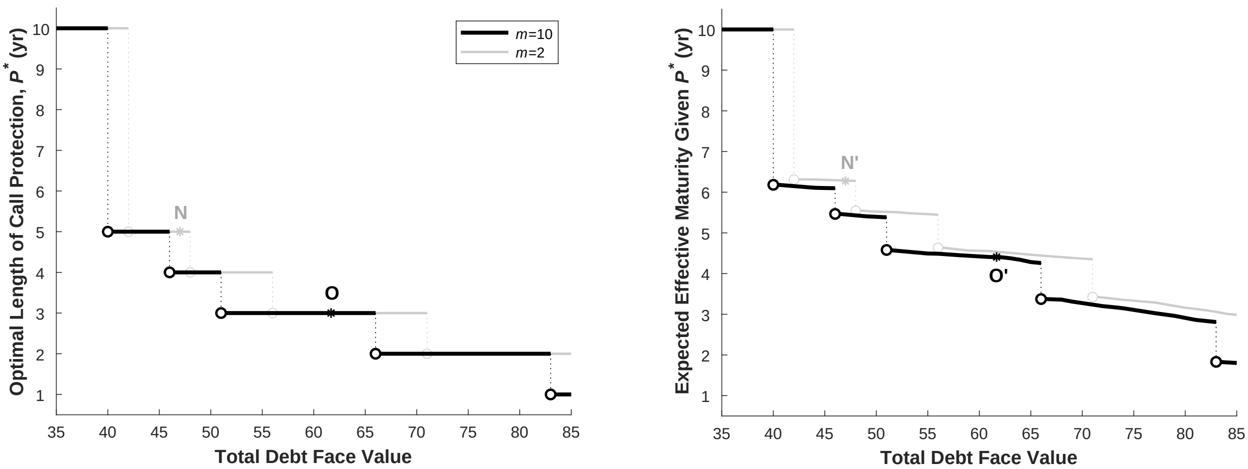
Figure 2: Modeling refinancing with callable bonds and rolling over non-callable bonds with lumpy debt maturity via tree and forest structure.



A. Choice of call protection length

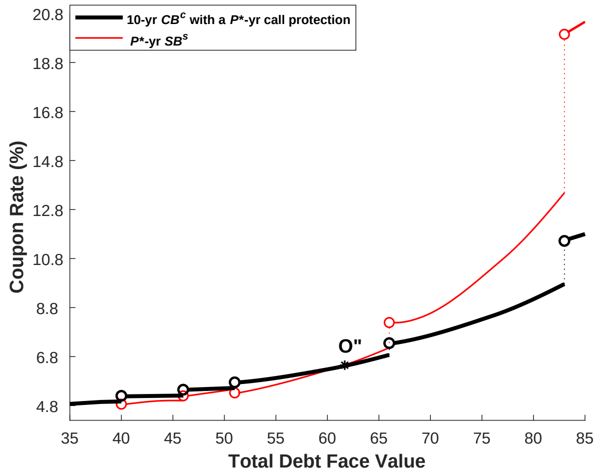
B. Expected effective maturity

Figure 3: The choice of call protection length and expected effective maturity. The x-axis in both panels represents the call protection length in years, which ranges from 1 to 10 years for a 10-year callable bond CB^c. The y-axis in Panel A represents the total levered firm value $V_0^{L.c}$ in Equation (2) when CB^c with a specific call protection length is issued and refinanced with an otherwise identical bond repeatedly. The y-axis in Panel B represents the time span in years for a call protection period, stated maturity, and the corresponding expected effective maturity. The star O in both panels refers to the choice of the call protection period that maximizes the total levered firm value. The O′ in Panel B refers to the expected effective maturity given that the protection length is optimally chosen. All other parameter values follow those in Table 5.

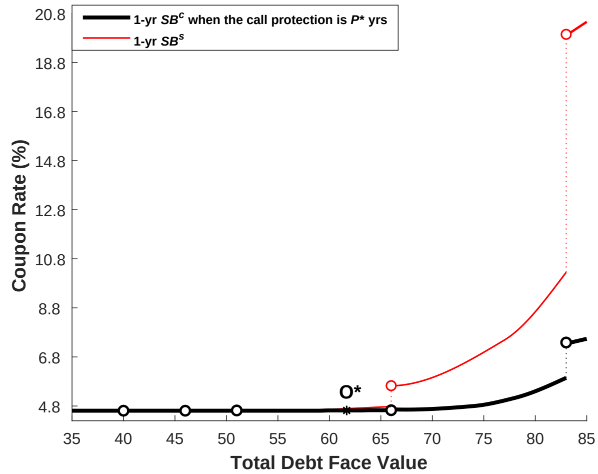


A. Optimal call protection length P^* B. Expected effective maturities given P^*

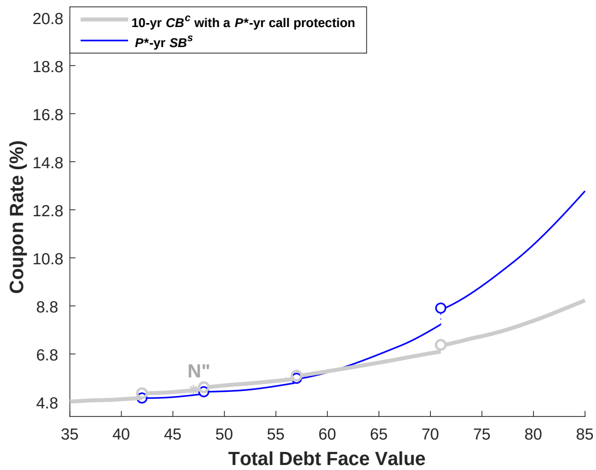
Figure 4: Optimal call protection length and expected effective maturities over total debt face values for two different levels of rollover frequency. In both panels, the x-axis represents the firm's total debt face value. The y-axis in Panels A and B represents the optimal lengths of call protection periods P^* and the corresponding expected effective maturities, respectively. The black lines refer to the scenario that the firm's debt structure consists of a 10-year callable bond CB^c and a 1-year (i.e., $m = 10$) non-callable bond SB^c. The gray lines indicate the scenario identical to the black lines in all other aspects except for the stated maturity of the SB^c, which equals 5 years (i.e., $m = 2$). The O and O′ in Panels A and B are those in Figure 3. The N and N′ refer to the scenario in which the total debt face value is 47, and the stated maturity of SB^c is 5 years. All other parameter values follow those in Table 5.



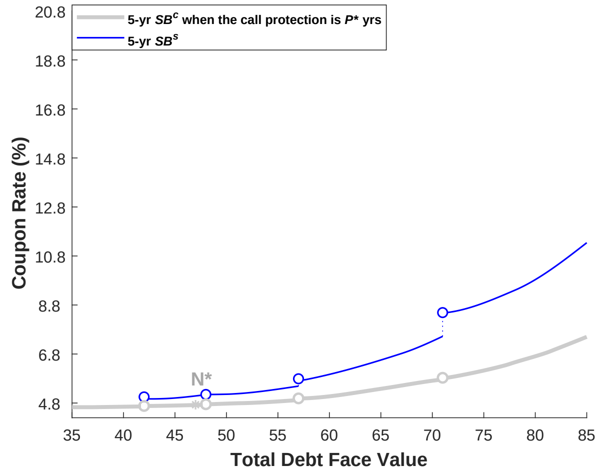
A. Coupon rates of 10-yr CB^c and P^*-yr SB^s as $m = 10$



B. Coupon rates of 1-yr SB^c and SB^s



C. Coupon rates of 10-yr CB^c and P^*-yr SB^s as $m = 2$



D. Coupon rates of 5-yr SB^c and SB^s

Figure 5: Interest costs on callable and non-callable bonds. The x and y-axes represent the total debt face value and the coupon rate for different two-bond debt structures. The debt structure comprises a $10/m$-year non-callable bond SB^c (in Panels B and D) and a 10-year callable bond CB^c with a P^*-year call protection (in Panels A and C), as defined in Equation (2), are denoted by black and gray colors for $m = 10$ and $m = 2$ scenarios, respectively. The optimal protection length P^* is determined to maximize the initial total levered firm value given different debt face values, as displayed in Panel A of Figure 4. The other debt structure comprises a $10/m$-year non-callable SB^s (in Panels B and D) and a P^*-year non-callable SB^s (in Panels A and C), as defined in Equation (3), are denoted by red and blue colors for $m = 10$ and $m = 2$ scenarios, respectively. Thus, the stated maturities of non-callables in Panels B and D are (10/10=) 1-year and (10/2=) 5-year, respectively. All other parameter values follow those in Table 5. The O$''$ and O* (N$''$ and N*) in Panels A and B (C and D) refer to the coupon rates of CB^c and SB^c in scenario O (N) in Panel A of Figure 4.

Table 1: Early Refinancing Activities Conducted by General Mills INC. and Barclay Bank PLC.

This table gives two examples of early refinancing activities. Panel A exhibits six callable bonds issued by General Mills INC. The latter four callables were issued close to the early redemption dates of the former two previously-issued callables. Panel B exhibits five callable bonds issued by Barclay Bank PLC. The latter two callables were issued just near the early redemption dates of the former three previously-issued callables.

<table>
<tr><td colspan="5">Panel A: General Mills INC.</td></tr>
<tr><td>Bond CUSIP</td><td>Offering Date</td><td>First Call Date</td><td>Call Effective Date</td><td>Maturity Date</td></tr>
<tr><td>37033LEY8</td><td>1998-02-05</td><td>2003-02-05</td><td>2003-02-05</td><td>2023-02-05</td></tr>
<tr><td>37033LFF8</td><td>1999-01-15</td><td>2003-01-22</td><td>2003-01-22</td><td>2011-01-22</td></tr>
<tr><td>37033EAX0</td><td rowspan="2">2003-01-31</td><td rowspan="4">2004-02-15</td><td rowspan="4">2004-02-15</td><td>2008-02-05</td></tr>
<tr><td>37033EAY8</td><td>2010-02-05</td></tr>
<tr><td>37033EAZ5</td><td rowspan="2">2003-02-07</td><td>2008-02-12</td></tr>
<tr><td>37033EBA9</td><td>2010-02-12</td></tr>
<tr><td colspan="5">Panel B: Barclay Bank PLC.</td></tr>
<tr><td>Bond CUSIP</td><td>Offering Date</td><td>First Call Date</td><td>Call Effective Date</td><td>Maturity Date</td></tr>
<tr><td>06738JCE2</td><td rowspan="3">2011-02-14</td><td rowspan="3">2012-02-17</td><td rowspan="3">2012-02-17</td><td>2024-02-17</td></tr>
<tr><td>06738JC93</td><td>2026-02-17</td></tr>
<tr><td>06738JD27</td><td>2031-02-17</td></tr>
<tr><td>06738JZ23</td><td rowspan="2">2012-02-10</td><td rowspan="2">2013-02-15</td><td rowspan="2">2013-02-15</td><td>2017-02-15</td></tr>
<tr><td>06738KL74</td><td>2022-02-15</td></tr>
</table>

Table 2: Bond characteristics summary.

This table reports summary statistics for our final bond sample. N and Stdev denote the number of bond samples and the standard deviation, respectively. Callable bonds are those with the flag CALLABLE = Y in Mergent FISD, and non-callable bonds are those with the flag CALLABLE = N. *BondStaM* (*BondCProt*) denotes the length of the stated bond maturity (call protection). *BondEffM* denotes the length of effective bond maturity. *BondElim* denotes the length of the time eliminated from the original bond's life due to early redemption. *BondCProtR* and *BondElimR* are two relative measures defined as *BondCProt*/*BondStaM* and *BondElim*/*BondStaM*, respectively. *BondCoupon* is the coupon rate for each bond. *Bond rating* is the score of the bond rating on the bond issue date. We follow the scores assigned in Mergent FISD (e.g, S&P's bond rating AAA = 1, and AA+ = 2). *Covenant count* is the number of restrictive covenants present in one bond. More details on variable definitions are shown in Appendix A.4.

Variable	N	Mean	Median	Stdev
Panel A: Callable bonds				
BondStaM (yrs)	41,670	12.29	10.01	8.98
BondEffM (yrs)	33,537	4.55	4.15	3.40
BondCProt(yrs)	41,646	3.77	2.98	5.34
BondCProtR	41,646	0.32	0.25	0.29
BondElim (yrs)	33,537	6.45	4.70	7.07
BondElimR	33,537	0.50	0.57	0.35
BondCoupon (%)	40,485	6.17	6.00	2.84
Offering amount($millions)	41,670	332.63	175.00	2,052.70
Bond rating	17,529	8.39	8.00	4.07
Covenant count	41,670	2.60	0.00	3.35
Panel B: Non-callable bonds				
BondStaM (yrs)	80,308	4.58	3.01	5.41
BondCoupon (%)	76,949	4.80	4.52	4.89
Offering amount($millions)	80,308	164.54	8.26	1,978.93
Bond rating	28,446	5.44	5.00	2.34
Covenant count	80,308	0.39	0.00	1.28

Table 3: Firm characteristics summary.

This table reports summary statistics for our firm-level data during the period 1990–2018. *Leverage* refers to the ratio of total assets to total stockholders' equity. *Curlia* denotes the ratio of debt in current liability to the sum of debt in current liability and debt due in more than one year. *M/B ratio* refers to market-to-book ratio. *Tangibility* refers to the ratio of tangible assets to total assets. *EBITDA* represents the ratio of earnings before interest, tax, depreciation, and amortization to total assets. *Cash* represents the ratio of cash and short-term investment to total assets. *Firm rating* refers to the ordinal score of S&P long-term firm credit rating. The rating score of a given year is computed using a conversion process in which AAA-rated firms are assigned a value of 1, and D-rated firms are assigned a value of 22. The definition of all variables on firm fundamentals are detailed in Table A.3 in Appendix A.4.

Variable	Firm-year Obs	Mean	Median	Stdev
Firm-level bond data				
FirmStaM (yrs)	46,812	11.81	10.00	6.80
FirmCProtR	26,560	0.51	0.49	0.23
FirmElimR	15,732	0.26	0.13	0.29
FirmCoupon (%)	48,082	7.11	7.04	2.64
Other firm fundamentals				
Total assets ($millions)	78,034	14,473.95	1,711.02	48,047.42
Leverage	78,015	3.91	2.61	6.99
Curlia	73,986	0.20	0.08	0.26
M/B ratio	65,491	1.75	1.34	1.21
Tangibility	75,090	0.33	0.26	0.28
EBITDA	75,050	0.10	0.11	0.13
Cash	77,945	0.12	0.05	0.16
Equity return	63,536	0.15	0.06	0.62
Firm rating	38,401	10.05	10.00	3.90

Table 4: Comparisons of bond characteristics between low-leverage and high-leverage (curlia) firms.

The comparison is performed via *BondStaM*, *BondEffM*, *BondCProt*, *BondCProtR*, *BondElim*, and *BondElimR*. The values represent the subsample averages of bond issuers in the first row of each panel. In particular, firms are classified as low-leverage (low-curlia) or high-leverage (high-curlia) in one year according to the median *Leverage* (*Curlia*) of the sample firms in that year in the sample period 1990–2018. The corresponding subsample represents the outstanding bonds issued by firms when they are classified as either low-leverage (low-curlia) or high-leverage (high-curlia) in that year. *, **, and *** denote that the difference in bond characteristics is statistically significantly at the 10%, 5% and 1% level, respectively.

Panel A: Low-leverage vs. high-leverage firms

	Low-leverage	High-leverage	Difference	t-value
BondStaM (yrs)	11.58	12.09	-0.51 ***	-3.16
BondEffM (yrs)	6.46	4.54	1.92 ***	9.92
BondCProt (yrs)	4.03	2.97	1.06 ***	22.77
BondCProtR	0.41	0.28	0.13 ***	34.41
BondElim (yrs)	6.46	7.55	-1.09 ***	-7.6
BondElimR	0.46	0.57	-0.11 ***	-14.09

Panel B: Low-curlia vs. high-curlia firms

	Low-curlia	High-curlia	Difference	t-value
BondStaM (yrs)	11.93	12.07	-0.14	-1.1
BondEffM (yrs)	5.2	4.35	0.85 ***	18.56
BondCProt (yrs)	4.02	2.7	1.32 ***	36.45
BondCProtR	0.39	0.26	0.13 ***	47.5
BondElim (yrs)	6.73	7.72	-0.99 ***	-8.69
BondElimR	0.47	0.55	-0.08 ***	-17.41

Table 5: Baseline parameters.

Market Condition	
Interest rate r	4.61%
Corporate tax rate τ	30.6%
Debt flotation cost γ	0.5%
Firm Characteristics	
Payout rate: q	2%
Firm value volatility σ	21%
Bankruptcy cost ω	37%
Debt Structure	
Stated maturity of the callable bond T	10
Stated maturity of the non-callable bond T/m	1
Rollover frequency m	10
Current fundamental V_0	100
Total debt face value $F_S + F_L$	61.68
Proportion of the non-callable bond's face value $F_S/(F_S + F_L)$	8%

Table 6: Early refinancing activities and changes in firm leverage.
In this table, we present a comparative analysis of leverage dynamics across three distinct phases for firms conducting early refinancing activities. In the columns with $D(EarlyRefinance) = 1$, we list the average leverage ratios before, during, and after the early refinancing year. In the columns with $D(EarlyRefinance) = 0$, we list the average leverage ratios during the same fiscal year for firms that did not conduct early refinancing activities. Difference represents the leverage ratio in the latter year minus that in the former year. We report the corresponding values of t-statistics and p-statistics adjusted for clustering at the firm level in the last two rows. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

	Leverage					
	$D(EarlyRefinance) = 1$			$D(EarlyRefinance) = 0$		
Before early refinance year	5.83		5.79	3.82		3.84
Early refinancing year		5.83	5.82		3.84	3.81
After early refinance year	5.71	5.72		3.80	3.81	
Difference	-0.11	-0.11	0.03	-0.02	-0.03	-0.03
t-value	-0.51	-0.50	0.14	-0.55	-0.73	-0.91
p-value	0.61	0.62	0.89	0.58	0.47	0.36

Table 7: Regressions of *FirmCProtR*.

This table reports regression results for the call protection length in different sub-samples. The dependent variable is *FirmCProtR*, which indicates the average proportion of call protection length to entire stated bond maturity; its definition is expressed in Equation (17). The independent variable *D(EarlyRefinance)* is a dummy variable which equals 1 when firms conduct an early refinancing activity. Other controls include *Termspread*, *ln(Assets)*, *Leverage*, *M/B Ratio*, *Tangibility*, *EBITDA*, *Cash*, and *Equity return*; their definitions are detailed in Appendix A.4. Three different sub-sample groups are considered based on three measures of rollover risk: leverage ratio (*Leverage*), level of debt in current liability (*Curlia*), and rollover intensity (*RI*) defined in Section 5.1. The observations at the firm-year level are classified as the high (low) rollover risk group in year t as either of their *Leverage*, *CurLia*, or *RI* is beyond (below) their medians in the same year. Firm fixed effects and year fixed effects are included. We report the value of t-statistics adjusted for clustering at the firm level in parentheses. *,**, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively.

	Dependent variable = *FirmCProtR*						
	(1) Full Sample	(2) High *Leverage*	(3) Low *Leverage*	(4) High *Curlia*	(5) Low *Curlia*	(6) High *RI*	(7) Low *RI*
D(EarlyRefinance)	-0.0275***	-0.0302***	-0.0095	-0.0381***	-0.0139***	-0.0374***	-0.0189***
	(-5.47)	(-4.97)	(-1.23)	(-4.17)	(-2.64)	(-4.34)	(-3.19)
Termspread	9.6064***	10.4401***	8.3523***	9.9880***	8.5311***	11.0100***	8.4814***
	(12.90)	(10.82)	(6.79)	(10.74)	(8.54)	(10.48)	(8.68)
ln(Asset)	0.0082	-0.0089	0.0196**	0.0059	0.0021	-0.0043	0.0067
	(1.36)	(-1.14)	(2.09)	(0.68)	(0.30)	(-0.45)	(0.92)
Leverage	0.0002	-0.0003	0.0001	0.0006	0.0002	0.0002	0.0001
	(1.24)	(-0.84)	(0.11)	(1.63)	(0.92)	(0.53)	(0.54)
M/B Ratio	0.0029	0.0075	0.0033	-0.0039	0.0052	-0.0089	0.0076*
	(0.74)	(1.48)	(0.59)	(-0.56)	(1.21)	(-1.18)	(1.67)
Tangibility	0.0435	0.0459	0.0524	-0.0975*	0.0985**	0.0012	0.0726*
	(1.14)	(0.80)	(1.01)	(-1.81)	(2.42)	(0.02)	(1.71)
EBITDA	-0.0018	0.0075	-0.0165	0.0633*	-0.0271	0.0372	-0.0339
	(-0.07)	(0.20)	(-0.47)	(1.65)	(-0.99)	(0.85)	(-1.06)
Cash	0.0033	-0.0131	0.0263	0.0011	-0.0112	-0.0025	-0.0082
	(0.10)	(-0.29)	(0.57)	(0.02)	(-0.32)	(-0.04)	(-0.22)
Equity Return	0.0060***	0.0068***	0.0015	0.0073**	0.0050**	0.0078**	0.0059**
	(3.29)	(2.78)	(0.48)	(2.25)	(2.28)	(2.35)	(2.44)
Constant	-9.1299***	-9.8256***	-7.9607***	-9.4048***	-8.0683***	-10.3856***	-8.0138***
	(-12.56)	(-10.41)	(-6.66)	(-10.44)	(-8.19)	(-10.25)	(-8.38)
Firm FEs	Y	Y	Y	Y	Y	Y	Y
Year FEs	Y	Y	Y	Y	Y	Y	Y
Adj. R-squared	0.6700	0.6739	0.7346	0.7285	0.6597	0.7232	0.6710
Observations	18967	11064	7291	7644	10446	6905	10774

Table 8: Regressions of *FirmElimR*.

This table reports regression results for the time span eliminated from the original bond's life due to early redemption in different sub-samples. The dependent variable is *FirmElimR*, which indicates the average proportion of the time span eliminated from the original bond's life due to bond redemption to entire stated bond maturity; its definition is expressed in Equation (18). Other controls include *Termspread*, *ln(Assets)*, *Leverage*, *M/B Ratio*, *Tangibility*, *EBITDA*, *Cash*, and *Equity return*; their definitions are detailed in Appendix A.4. Three different sub-sample groups are considered based on three measures of rollover risk: leverage ratio (*Leverage*), level of debt in current liability (*Curlia*), and rollover intensity (*RI*) defined in Section 5.1. The observations at the firm-year level are classified as the high (low) rollover risk group in year t as either of their *Leverage*, *CurLia*, or *RI* is beyond (below) their medians in the same year. Firm fixed effects and year fixed effects are included. We report the value of t-statistics adjusted for clustering at the firm level in parentheses. *,**, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively.

	Dependent variable = *FirmElimR*						
	(1) Full Sample	(2) High *Leverage*	(3) Low *Leverage*	(4) High *Curlia*	(5) Low *Curlia*	(6) High *RI*	(7) Low *RI*
$D(EarlyRefinance)$	0.1227***	0.1299***	0.1129***	0.1759***	0.0731***	0.1754***	0.0869***
	(15.16)	(12.76)	(8.14)	(14.55)	(6.46)	(14.14)	(8.23)
Termspread	-6.4206***	-5.1390***	-8.7273***	-5.1260***	-9.4587***	-5.7323***	-7.0134***
	(-5.24)	(-3.36)	(-3.83)	(-3.20)	(-4.39)	(-2.95)	(-4.33)
ln(Asset)	-0.0243***	-0.0237**	-0.0124	-0.0250**	-0.0220*	-0.0363***	-0.0319**
	(-2.96)	(-2.15)	(-0.86)	(-2.13)	(-1.82)	(-3.03)	(-2.52)
Leverage	-0.0010**	-0.0012	0.0002	-0.0011	-0.0011*	-0.0015*	-0.0008
	(-2.06)	(-1.55)	(0.13)	(-1.53)	(-1.78)	(-1.91)	(-1.26)
M/B Ratio	-0.0163**	-0.0119	-0.0155*	-0.0141	-0.0132	-0.0332***	-0.0066
	(-2.41)	(-1.13)	(-1.68)	(-1.42)	(-1.16)	(-3.04)	(-0.63)
Tangibility	-0.0438	-0.0321	-0.0816	0.0063	-0.1187*	-0.0503	-0.0926
	(-0.93)	(-0.57)	(-1.00)	(0.10)	(-1.78)	(-0.71)	(-1.26)
EBITDA	0.0663	0.0615	0.0329	0.1268	0.0348	0.1468	0.1270
	(1.28)	(0.82)	(0.39)	(1.40)	(0.49)	(1.59)	(1.63)
Cash	0.1735***	0.1567**	0.0896	0.1859**	0.1671*	0.2053***	0.1782**
	(3.32)	(2.25)	(1.11)	(2.43)	(1.87)	(2.68)	(2.10)
Equity Return	0.0085	0.0079	0.0046	0.0157	0.0003	0.0202**	0.0031
	(1.41)	(1.05)	(0.42)	(1.60)	(0.04)	(2.07)	(0.33)
Constant	6.9405***	5.6344***	9.1959***	5.5579***	10.0758***	6.3549***	7.6125***
	(5.67)	(3.69)	(4.05)	(3.49)	(4.66)	(3.25)	(4.75)
Firm FEs	Y	Y	Y	Y	Y	Y	Y
Year FEs	Y	Y	Y	Y	Y	Y	Y
Adj. R-squared	0.4347	0.4679	0.4267	0.4272	0.3926	0.4285	0.4349
Observations	10628	6339	3714	5737	4131	4995	4690

Table 9: Regressions of *FirmCoupon*.

This table reports regression results for the interest disbursement on firms' outstanding bonds in different sub-samples. The dependent variable is *FirmCoupon*, which indicates the average interest disbursement on firms' outstanding corporate bonds; its definition is expressed in Equation (16). Other controls include *Termspread*, *ln(Assets)*, *Leverage*, *M/B Ratio*, *Tangibility*, *EBITDA*, *Cash*, and *Equity return*; their definitions are detailed in Appendix A.4. Three different sub-sample groups are considered based on three measures of rollover risk: leverage ratio (*Leverage*), level of debt in current liability (*Curlia*), and rollover intensity (*RI*) defined in Section 5.1. The observations at the firm-year level are classified as the high (low) rollover risk group in the year t as either of their *Leverage*, *CurLia*, or *RI* is beyond (below) their medians in the same year. Firm fixed effects and year fixed effects are included. We report the value of t-statistics adjusted for clustering at the firm level in parentheses. *,**, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively.

	Dependent variable = *FirmCoupon*						
	(1) Full Sample	(2) High *Leverage*	(3) Low *Leverage*	(4) High *Curlia*	(5) Low *Curlia*	(6) High *RI*	(7) Low *RI*
$D(EarlyRefinance)$	-0.0240	-0.0482	0.0251	-0.0755	-0.0147	-0.1108*	0.0232
	(-0.59)	(-0.86)	(0.52)	(-0.96)	(-0.40)	(-1.75)	(0.47)
Termspread	-97.2879***	-108.2460***	-77.1650***	-110.3911***	-85.5454***	-96.3923***	-95.8045***
	(-21.89)	(-18.70)	(-10.49)	(-18.75)	(-13.64)	(-14.63)	(-17.12)
ln(Asset)	-0.4421***	-0.4256***	-0.4681***	-0.4180***	-0.4571***	-0.4338***	-0.4293***
	(-9.95)	(-7.51)	(-7.82)	(-7.33)	(-7.91)	(-6.85)	(-7.84)
Leverage	0.0036**	0.0103***	-0.0041	0.0043*	0.0031*	0.0041*	0.0048**
	(2.41)	(3.60)	(-0.69)	(1.92)	(1.67)	(1.66)	(2.53)
M/B Ratio	-0.1608***	-0.1726***	-0.1361***	-0.1598***	-0.1723***	-0.1600***	-0.1973***
	(-6.34)	(-4.44)	(-4.27)	(-3.99)	(-5.54)	(-3.62)	(-6.33)
Tangibility	-0.3967*	-0.5887*	0.0764	-0.5979*	-0.1856	-0.3857	-0.1556
	(-1.73)	(-1.67)	(0.23)	(-1.78)	(-0.62)	(-1.11)	(-0.51)
EBITDA	-0.1154	-0.0161	0.0473	0.0700	-0.2364	0.3426	-0.3592
	(-0.67)	(-0.07)	(0.19)	(0.27)	(-1.00)	(1.13)	(-1.42)
Cash	-0.7402***	-0.5833**	-0.6494**	-0.1767	-0.9319***	-0.5381*	-0.8231***
	(-3.77)	(-1.96)	(-2.51)	(-0.61)	(-3.51)	(-1.77)	(-3.32)
Equity Return	0.1281***	0.1324***	0.1253***	0.1235***	0.1274***	0.1019***	0.1704***
	(8.89)	(6.58)	(4.79)	(4.87)	(6.45)	(4.44)	(7.79)
Constant	108.8851***	120.0057***	88.1917***	121.9777***	97.0241***	107.8474***	107.2447***
	(25.03)	(21.38)	(12.08)	(21.27)	(15.64)	(16.57)	(19.48)
Firm FEs	Y	Y	Y	Y	Y	Y	Y
Year FEs	Y	Y	Y	Y	Y	Y	Y
Adj. R-squared	0.7967	0.8032	0.8168	0.7894	0.8104	0.7995	0.8012
Observations	35805	20047	15026	17118	17440	14733	19215

Appendix A

A.1 Backward-Recursive Pricing Algorithm Using a Forest

A.1.1 Backward Induction Using the Forest in Panel C of Figure 2: A Step-by-Step Guide

In this paper, we employ a forest to evaluate equity and debt when calls for early debt refinancing are limited to specified call dates. The forest is composed of M layers of CRR binomial trees with n equal-length time steps Δt, $\Delta t = T/n$, and the parameters:

$$u = e^{\sigma\sqrt{\Delta t}}, \quad d = e^{-\sigma\sqrt{\Delta t}},$$
$$P_u = \frac{e^{(r-q)\Delta t} - d}{u-d}, \quad P_d = 1 - P_u,$$

to discretely simulate the firm value process of Equation (1) over different statuses of debt structure. The variables u and d parameterize the state of the firm value, from the initial value V either up to Vu or down to Vd at the next time step; P_u and P_d parameterize the probability of up and down movement of the firm value for each time step. To evaluate the $T/2$-year $SB^c_{T/2}$ and the T-year $CB^c_{T/2,T}$ with a single call date at $t = T/2$ using a $2T$-year time span, we apply backward induction in the forest with $M = 4$ and $n = 6$ in Panel C of Figure 2 to capture all possible changes in debt structure due to debt (early) refinancing. We detail here this backward induction procedure step-by-step as follows.

To assess the gains and losses from activities of debt refinancing, the values of other later-issued bonds should be computed. We note that the previously-issued and later-issued bonds are identical in all other aspects except for their issue dates. As illustrated in Figure A.1, the first two later-issued bonds are $T/2$-year SB^c_T and T-year $CB^c_{T,3T/2}$ (i.e., the bonds in the dark red piece of the second layer); they are issued at $t = T/2$ to repay the maturing $SB^c_{T/2}$ and early redeem $CB^c_{T/2,T}$ (i.e., the bonds in the black piece of the first layer), respectively. The second two bonds are $T/2$-year $SB^c_{3T/2}$ and T-year

$CB^c_{3T/2,\,2T}$ (i.e., the bonds in the dark green piece of the third layer); they are issued at $t = T$ to repay the maturing SB^c_T and $CB^c_{T/2,\,T}$ (i.e., the bonds in the gray piece of the first layer), or to repay the maturing SB^c_T and early redeem $CB^c_{T,\,3T/2}$, respectively. The third two bonds are the two $T/2$-year SB^s_{2T}; C_S and C_L (F_S and F_L) are their coupon rates (face values) of the two bonds. They are issued at $t = 3T/2$ to repay the maturing $SB^c_{3T/2}$ and $CB^c_{T,\,3T/2}$ (i.e., the bonds in the light red piece of the second layer), or to repay the maturing $SB^c_{3T/2}$ and early redeem $CB^c_{3T/2,\,2T}$.

The backward induction procedure in the forest starts from $t = 2T$ and are separated into nine stages. In each stage, we work backward in the tree from the lowest layer to the highest one as follows.

Stage 1: $t = 2T$

As illustrated in Figure A.1, there are two statues of debt structure. The 4^{th} **Layer** refers to the first one comprised of two $T/2$-year SB^s_{2T}. The 3^{rd} **Layer** refers to the second one comprised of a $T/2$-year SB^c_{2T} and a T-year $CB^c_{3T/2,\,2T}$. We then employ two CRR trees; each simulates the firm value evolution over one state of debt structure.

Stage 1 – 4^{th} Layer:

If the firm chooses to early redeem the $CB^c_{3T/2,\,2T}$ and simultaneously repay the maturing $SB^c_{3T/2}$ at $t = 3T/2$, the two SB^s_{2T} will be issued. Thus at $t = 2T$, the capital structure components are the two SB^s_{2T} and the equity $E^s_{2T,\,2T}$. The equity values for the terminal nodes of the fourth layer orange tree in Panel C are:

$$E^s_{aT,bT}(V_t, t) = max\left(V_t + \delta_t - (1-\tau)(C_S F_S + C_L F_L)\Delta t - (F_S + F_L)\,,\,0\right), \qquad (22)$$

in which $a = b = 2$. The δ_t is set to $V_t e^{q\Delta t} - V_t$ and will converge to $qV_t dt$ if Δt is small enough. The values of the corresponding non-callable bond is:

$$SB^s_{aT}(V_t, t \,|\, T/2) = \begin{cases} F_{\mathbb{M}} + C_{\mathbb{M}} F_{\mathbb{M}} \Delta t & \text{if } E^s_{aT,bT}(V_t, t) > 0, \\ (1-\omega)(V_t + \delta_t) \times \alpha_{\mathbb{M}} & \text{otherwise,} \end{cases} \tag{23}$$

in which $\mathbb{M}$ can be S or L; the $\alpha_{\mathbb{M}}$ equals $F_{\mathbb{M}}/(F_S + F_L)$ since the two non-callable bonds are equally senior during the liquidation process.

Stage 1 – 3^{rd} Layer:

If the firm repays the maturing $SB^c_{3T/2}$ alone through the proceeds from raising SB^c_{2T} at $t = 3T/2$, then the three capital structure components at $t = 2T$ are SB^c_{2T}, $CB^c_{3T/2,\,2T}$, and $E^c_{2T,\,2T}$. The equity values for the terminal nodes of the third layer green tree are:

$$E^c_{aT,bT}(V_t, t) = max\left(V_t + \delta_t - (1-\tau)(C_S F_S + C_L F_L)\Delta t - (F_S + F_L)\,,\, 0\right), \tag{24}$$

The values of the corresponding SB^c_{2T} and $CB^c_{3T/2,\,2T}$ are:

$$SB^c_{aT}(V_t, t \,|\, T/2) = \begin{cases} F_S + C_S F_S \Delta t & \text{if } E^c_{aT,bT}(V_t, t) > 0, \\ (1-\omega)(V_t + \delta_t) \times \alpha_S & \text{otherwise,} \end{cases} \tag{25}$$

$$CB^c_{pT,bT}(V_t, t \,|\, T/2, T) = \begin{cases} F_L + C_L F_L \Delta t & \text{if } E^c_{aT,bT}(V_t, t) > 0, \\ (1-\omega)(V_t + \delta_t) \times \alpha_L & \text{otherwise,} \end{cases} \tag{26}$$

in which $a = b = 2$ and $p = 3/2$; α_S and α_L are equal to $F_S/(F_S + F_L)$ and $F_L/(F_S + F_L)$, respectively.

Stage 2: $3T/2 < t < 2T$

As illustrated in Figure A.1, the statues of debt structure are identical to those in Stage 1.

Stage 2 – 4^{th} **Layer:**

When the debt structure components are two SB^s_{2T}, the equity value is expressed as:

$$E^s_{aT,bT}(V_t,t) = max\left(\delta_t - (1-\tau)(C_SF_S + C_LF_L)\Delta t + \underbrace{E^s_{aT,bT}(V_{t^+},t^+)}_{\text{A1}}, 0\right), \tag{27}$$

in which $a = b = 2$. The term A1 is the expected present equity value right after time t, and its value can be calculated using backward induction in the fourth layer orange CRR tree as follows:

$$e^{-r\Delta t}\Big(P_u \times E^s_{aT,bT}(V_tu, t+\Delta t) + P_d \times E^s_{aT,bT}(V_td, t+\Delta t)\Big). \tag{28}$$

The value of SB^s_{2T} is:

$$SB^s_{aT}(V_t, t\,|\,T/2) = \begin{cases} C_{\mathbb{M}}F_{\mathbb{M}}\Delta t + \underbrace{SB^s_{aT}(V_{t^+}, t^+\,|\,T/2)}_{\text{A2}} & \text{if } E^s_{aT,bT}(V_t,t) > 0, \\ (1-\omega)(V_t+\delta_t) \times \alpha_{\mathbb{M}} & \text{otherwise,} \end{cases} \tag{29}$$

in which the term A2 refers to the expected present bond value when the coupon payment occurred at time t is not yet considered; its value can also be calculated using backward induction in the same tree as follows:

$$e^{-r\Delta t}\Big(P_u \times SB^s_{aT}(V_tu, t+\Delta t\,|\,T/2) + P_d \times SB^s_{aT}(V_td, t+\Delta t\,|\,T/2)\Big). \tag{30}$$

Stage 2 – 3^{rd} **Layer:**

When the debt structure components are SB^c_{2T} and $CB^c_{3T/2,\,2T}$, the equity value is in turn expressed as:

$$E^c_{aT,bT}(V_t,t) = max\left(\delta_t - (1-\tau)(C_S F_S + C_L F_L)\Delta t + \underbrace{E^c_{aT,bT}(V_{t^+},t^+)}_{\text{A3}}, 0\right), \tag{31}$$

in which $a = b = 2$. The term A3 is the expected present equity value right after time t, and its value can be calculated using backward induction in the third layer green CRR tree as follows:

$$e^{-r\Delta t}\Big(P_u \times E^c_{aT,bT}(V_t u, t+\Delta t) + P_d \times E^c_{aT,bT}(V_t d, t+\Delta t)\Big). \tag{32}$$

The value of SB^c_{2T} is:

$$SB^c_{aT}(V_t,t\,|\,T/2) = \begin{cases} C_S F_S \Delta t + \overbrace{SB^c_{aT}(V_{t^+},t^+\,|\,T/2)}^{\text{A4}} & \text{if } E^c_{aT,bT}(V_t,t) > 0, \\ (1-\omega)(V_t+\delta_t)\times\alpha_S & \text{otherwise,} \end{cases} \tag{33}$$

in which the term A4 can be evaluated using backward induction in the same tree as follows:

$$e^{-r\Delta t}\Big(P_u \times SB^c_{aT}(V_t u, t+\Delta t\,|\,T/2) + P_d \times SB^c_{aT}(V_t d, t+\Delta t\,|\,T/2)\Big). \tag{34}$$

The value of $CB^c_{3T/2,2T}$ is:

$$CB^c_{pT,bT}(V_t,t\,|\,T/2,T) = \begin{cases} C_L F_L \Delta t + \overbrace{CB^c_{pT,bT}(V_{t^+},t^+\,|\,T/2,T)}^{\text{A5}} & \text{if } E^c_{aT,bT}(V_t,t) > 0, \\ (1-\omega)(V_t+\delta_t)\times\alpha_L & \text{otherwise,} \end{cases} \tag{35}$$

in which $p = 3/2$. The term A5 can also be evaluated using backward induction in the same tree as follows:

$$e^{-r\Delta t}\Big(P_u \times CB^c_{pT,bT}(V_t u, t+\Delta t\,|\,T/2,T) + P_d \times CB^c_{pT,bT}(V_t d, t+\Delta t\,|\,T/2,T)\Big). \tag{36}$$

Stage 3: $t = 3T/2$

As illustrated in Figure A.1, there are three possible scenarios. The 4^{th} **Layer** denotes the scenario in which the firm issues the two SB_{2T}^{s} to repay the maturing $SB_{3T/2}^{c}$ and early redeem $CB_{3T/2,\,2T}^{c}$ (i.e., bonds in the third layer), or to repay the maturing $SB_{3T/2}^{c}$ and $CB_{T,\,3T/2}^{c}$ (i.e., bonds in the second layer). The 3^{rd} **Layer** refers to the scenario in which the firm will either choose to refinance $CB_{3T/2,\,2T}^{c}$ early and simultaneously roll over $SB_{3T/2}^{c}$ through the proceeds from raising the two SB_{2T}^{s} (i.e., transfer from the third layer to the fourth), roll over $SB_{3T/2}^{c}$ alone through the proceeds from raising SB_{2T}^{c} (i.e., stay in the third layer), or declare default. Finally, the 2^{nd} **Layer** represents the scenario in which the firm will either repay the maturing $SB_{3T/2}^{c}$ and $CB_{T,\,3T/2}^{c}$ through the proceeds from raising SB_{2T}^{s} (i.e., transfer from the second layer to the fourth) or declare default.

Stage 3 – 4^{th} **Layer:**

When the firm issues the two SB_{2T}^{s} at $t = 3T/2$, the equity value on the issue date can be determined using backward induction in the fourth layer orange CRR tree as the expression in Equation (28). The value of SB_{2T}^{s} can also be computed using the expression in Equation (30).

Stage 3 – 3^{rd} **Layer:**

The firm will either choose to refinance $CB_{3T/2,\,2T}^{c}$ early and simultaneously roll over the $SB_{3T/2}^{c}$ through the proceeds from raising the two SB_{2T}^{s}, roll over $SB_{3T/2}^{c}$ alone through the proceeds from raising the SB_{2T}^{c}, or declare default. Since the firm will make its default and early refinancing decision to serve shareholders' best interests, the equity value is expressed in three pieces as follows:

$$
\begin{aligned}
&E^c_{aT,bT}(V_t,t) \\
&= max\Bigg(\underbrace{E^s_{(a+\frac{1}{2})T,bT}(V_t,t)}_{\substack{\text{levered equity value}\\ \text{when two } SB^s_{2T}\\ \text{are outstanding}}} + \delta_t - (1-\tau)(C_SF_S + C_LF_L)\Delta t \\
&\qquad \overbrace{\underbrace{-(F_S+K_t)}_{\substack{\text{repayment of}\\ SB^c_{3T/2} \text{ and } CB^c_{3T/2,2T}}} \quad \underbrace{+(1-\gamma)\Big(SB^s_{bT}(V_t,t\,|\,T/2) + SB^s_{bT}(V_t,t\,|\,T/2)\Big)}_{\substack{\text{proceeds from raising}\\ \text{the two new } SB^s_{2T}}}}^{\text{gain or loss from rollover and early refinancing}}, \\
&\qquad \underbrace{E^c_{(a+\frac{1}{2})T,bT}(V_t,t)}_{\substack{\text{levered equity value when}\\ SB^c_{2T} \text{ and } CB^c_{3T/2,\,2T}\\ \text{are outstanding}}} \ + \delta_t - (1-\tau)(C_SF_S + C_LF_L)\Delta t \\
&\qquad \overbrace{\underbrace{-F_S}_{\substack{\text{repayment of}\\ \text{the maturing}\\ SB^c_{3T/2}}} \quad \underbrace{+(1-\gamma)SB^c_{(a+\frac{1}{2})T}(V_t,t\,|\,T/2)}_{\substack{\text{proceeds from raising}\\ \text{the new } SB^c_{2T}}}}^{\text{rollover gain or loss}}, \ \underbrace{0}_{\text{default}}\Bigg).
\end{aligned}
\tag{37}
$$

in which $a = 3/2$ and $b = 2$. We note that the K_t in the first piece of above equity value is the scheduled call price, and the values of the two orange terms are the final evaluation results in Stage $3-4^{th}$ **Layer**. In addition, based on the final evaluation results in Stage $2-3^{rd}$ **Layer**, the values of the light green terms in the second piece of the equity value are calculated using the expressions in Equations (32) and (34), which denote the backward induction in the third layer light green tree. On the other hand, the value of the maturing $SB^c_{3T/2}$ is expressed by Equation (25), and the $CB^c_{3T/2,2T}$ value is:

$$
CB^c_{pT,bT}(V_t,t\,|\,T/2,T) = \begin{cases} C_LF_L\Delta t + K_t & \text{if } E^c_{aT,bT}(V_t,t) > 0 \\ & \text{and call is announced} \\ C_LF_L\Delta t + \underbrace{CB^c_{pT,bT}(V_{t^+},t^+\,|\,T/2,T)}_{\text{A6}} & \text{if } E^c_{aT,bT}(V_t,t) > 0 \\ & \text{and call is not announced} \\ (1-\omega)(V_t+\delta_t)\times\alpha_L & \text{otherwise,} \end{cases} \tag{38}
$$

in which p is 3/2 and α_L equals $F_L/(F_S + F_L)$. The term A6 can be evaluated using the expression in Equation (36), which denotes the backward induction in the same light green tree.

Stage 3 $-$ 2^{nd} **Layer:**

The firm will either roll over the maturing $SB^c_{3T/2}$ and $CB^c_{T,3T/2}$ by issuing the two SB^s_{2T}, or announce default. Since the firm will make its default decision to serve shareholders' best interests, the equity value is expressed as:

$$
\begin{aligned}
&E^c_{aT,bT}(V_t,t)\\
&= max\Bigg(\underbrace{E^s_{(a+\frac{1}{2})T,(b+\frac{1}{2})T}(V_t,t)}_{\substack{\text{levered equity value when}\\ \text{the two } SB^s_{2T} \text{ are outstanding}}} + \delta_t - (1-\tau)(C_S F_S + C_L F_L)\Delta t \\
&\qquad \overbrace{\underbrace{-(F_S+F_L)}_{\substack{\text{repayment of}\\ \text{the maturing}\\ SB^c_{3T/2} \text{ and } CB^c_{T,3T/2}}} \underbrace{+(1-\gamma)\Big(SB^s_{(a+\frac{1}{2})T}(V_t,t\,|\,T/2) + SB^s_{(a+\frac{1}{2})T}(V_t,t\,|\,T/2)\Big)}_{\text{proceeds from raising two new } SB^s_{2T}}}^{\text{rollover gain and loss}}, \underbrace{0}_{\text{default}} \Bigg).
\end{aligned}
\tag{39}
$$

in which $a = b = 3/2$. The values of the two orange terms in this equation are the final evaluation results in Stage 3$-$$4^{th}$ **Layer**. The corresponding values of the $SB^c_{3T/2}$ and $CB^c_{T,3T/2}$ can be expressed by Equations (25) and (26), for which $p = 1$.

Stage 4: $T < t < 3T/2$

Like Stage 2, there are also two statues of debt structure, as illustrated in Figure A.1. The 3^{rd} **Layer** refers to the one comprised of a $T/2$-year $SB^c_{3T/2}$ and a T-year $CB^c_{3T/2,2T}$. In this scenario, the values of the equity and the two bonds are expressed as Equations (31), (33), and (35), in which $a = 3/2$, $b = 2$, and $p = 3/2$. The 2^{nd} **Layer** refers to the one comprised of a $T/2$-year $SB^c_{3T/2}$ and a T-year $CB^c_{T,3T/2}$. The values of the equity and the two bonds are also expressed by the same three equations; however, $a = b = 3/2$, and

$p = 1$.

Stage 5: $t = T$

Like Stage 3, there are also three possible scenarios, as illustrated in Figure A.1. The 3^{rd} **Layer** denotes the scenario in which the firm issues $SB^c_{3T/2}$ and $CB^c_{3T/2,\,2T}$ to repay the maturing SB^c_T and early redeem $CB^c_{T,\,3T/2}$ (i.e., bonds in the second layer), or to repay the maturing SB^c_T and $CB^c_{T/2,\,T}$ (i.e., bonds in the first layer). On the other hand, the 2^{nd} **Layer** refers to the scenario in which the firm will either choose to refinance $CB^c_{T,\,3T/2}$ early and simultaneously roll over the SB^c_T through the proceeds from raising $CB^c_{3T/2,\,2T}$ and $SB^c_{3T/2}$ (i.e., transfer from the second layer to the third), roll over SB^c_T alone through the proceeds from raising $SB^c_{3T/2}$ (i.e., stay in the second layer), or declare default. Finally, the 1^{st} **Layer** represents the scenario in which the firm will either repay the maturing SB^c_T and $CB^c_{T/2,\,T}$ through the proceeds from raising $SB^c_{3T/2}$ and $CB^c_{3T/2,\,2T}$ (i.e., transfer from the first layer to the third) or declare default.

Stage 5 – 3^{rd} Layer:

When the firm issues $SB^c_{3T/2}$ and $CB^c_{3T/2,2T}$ at $t = T$, the equity value on the issue date can be determined using backward induction in the third layer green CRR tree, as described in Equation (32), in which $a = 3/2$ and $b = 2$. The values of $SB^c_{3T/2}$ and $CB^c_{3T/2,\,2T}$ can also be computed using the expressions of Equations (34) and (36), for which $p = 3/2$.

Stage 5 – 2^{nd} Layer:

The firm will either choose to refinance $CB^c_{T,\,3T/2}$ early and simultaneously roll over the SB^c_T through the proceeds from raising $SB^c_{3T/2}$ and $CB^c_{3T/2,\,2T}$, roll over SB^c_T alone through the proceeds from raising $SB^c_{3T/2}$, or declare default. Again, since the firm will make its default and early refinancing decision to serve shareholders' best interests, the equity value is expressed in three pieces as follows:

$$
\begin{aligned}
&E^c_{aT,bT}(V_t,t)\\
&= max\Bigg(\underbrace{E^c_{(a+\frac{1}{2})T,(b+\frac{1}{2})T}(V_t,t)}_{\substack{\text{levered equity value when}\\ SB^c_{3T/2}\text{ and }CB^c_{3T/2,2T}\\ \text{are outstanding}}} + \delta_t - (1-\tau)(C_S F_S + C_L F_L)\Delta t\\
&\qquad \overbrace{\underbrace{-(F_S+K_t)}_{\substack{\text{repayment of}\\ SB^c_T\text{ and }CB^c_{T,3T/2}}} \underbrace{+(1-\gamma)\Big(SB^c_{(a+\frac{1}{2})T}(V_t,t\,|\,T/2) + CB^c_{(p+\frac{1}{2})T,(b+\frac{1}{2})T}(V_t,t\,|\,T/2,T)\Big)}_{\substack{\text{proceeds from raising the new}\\ SB^c_{3T/2}\text{ and }CB^c_{3T/2,2T}}}}^{\text{gain or loss from rollover and early refinancing}},\\
&\qquad \underbrace{E^c_{(a+\frac{1}{2})T,bT}(V_t,t)}_{\substack{\text{levered equity value when}\\ SB^c_{3T/2}\text{ and }CB^c_{T,3T/2}\\ \text{are outstanding}}} + \delta_t - (1-\tau)(C_S F_S + C_L F_L)\Delta t)\\
&\qquad \overbrace{\underbrace{-F_S}_{\substack{\text{repayment of}\\ \text{the maturing}\\ SB^c_T}} \underbrace{+(1-\gamma)SB^c_{(a+\frac{1}{2})T}(V_t,t\,|\,T/2)}_{\substack{\text{proceeds from raising}\\ \text{the new }SB^c_{3T/2}}}}^{\text{rollover gain or loss}},\ \underbrace{0}_{\text{default}}\Bigg).
\end{aligned}
\tag{40}
$$

in which $a = 1$, $b = 3/2$, and $p = 1$. The values of the green terms in the first piece of the equity value are the final evaluation results in Stage $5-3^{rd}$ **Layer**. In addition, based on the final evaluation results in Stage $4-2^{nd}$ **Layer**, the values of the light red terms in the second piece of the equity value are calculated using the expressions in Equations (32) and (34), which denote the backward induction in the second layer red tree. On the other hand, the corresponding value of the maturing SB^c_T is expressed by Equation (25), and the value of the $CB^c_{T,3T/2}$ value is expressed by Equation (38).

Stage $5-1^{st}$ Layer:

The firm will roll over the maturing SB^c_T and $CB^c_{T/2,T}$ by issuing $SB^c_{3T/2}$ and $CB^c_{3T/2,2T}$, or announce default. Since the firm will make its default decision to serve shareholders' best interests, the equity value is expressed as:

$$
\begin{aligned}
&E^c_{aT,bT}(V_t,t)\\
&= max\Bigg(\underbrace{E^c_{(a+\frac{1}{2})T,(b+1)T}(V_t,t)}_{\substack{\text{levered equity value when}\\ SB^c_{3T/2}\text{ and }CB^c_{3T/2,2T}\\ \text{are outstanding}}} + \delta_t - (1-\tau)(C_S F_S + C_L F_L)\Delta t\\
&\qquad \overbrace{\underbrace{-(F_S+F_L)}_{\substack{\text{repayment of}\\ \text{the maturing}\\ SB^c_T \text{ and } CB^c_{T/2,T}}} \underbrace{+(1-\gamma)\Big(SB^c_{(a+\frac{1}{2})T}(V_t,t\,|\,T/2) + CB^c_{(p+1)T,(b+1)T}(V_t,t\,|\,T/2,T)\Big)}_{\substack{\text{proceeds from raising the new}\\ SB^c_{3T/2}\text{ and }CB^c_{3T/2,2T}}}}^{\text{rollover gain and loss}},\ \underbrace{0}_{\text{default}}\Bigg),
\end{aligned}
\tag{41}
$$

in which $a = b = 1$ and $p = 1/2$. The values of the green terms are the final evaluation results in Stage $5-3^{rd}$ **Layer**. The corresponding values of the maturing SB^c_T and $CB^c_{T/2,T}$ can be expressed by Equations (25) and (26).

Stage 6: $T/2 < t < T$

Like Stages 2 and 4, there are two statues of debt structure, as illustrated in Figure A.1. The 2^{nd} **Layer** refers to the one comprised of a $T/2$-year SB^c_T and a T-year $CB^c_{T,3T/2}$. The values of the equity and the two bonds are expressed by Equations (31), (33), and (35), in which $a = 1$, $b = 3/2$, and $p = 1$. On the other hand, the 1^{st} **Layer** refers to the one comprised of a $T/2$-year SB^c_T and a T-year $CB^c_{T/2,T}$. The values of the equity and the two bonds are also expressed by the same three equations; however, $a = b = 1$, and $p = 1/2$.

Stage 7: $t = T/2$

Unlike Stages 3 and 5, which have three possible scenarios, this stage has only two, as illustrated in Figure A.1. The 2^{nd} **Layer** denotes the scenario in which the firm issues SB^c_T and $CB^c_{T,3T/2}$ at $t = T/2$ to repay the maturing $SB^c_{T/2}$ and early redeem $CB^c_{T/2,T}$

(i.e., bonds in the first layer). The equity value on the issue date can be determined using backward induction in the 2rd layer CRR tree, as described by Equation (32). The values of SB^c_T and $CB^c_{T,3T/2}$ can also be computed using the expressions of Equations (34) and (36), in which $a = 1$, $b = 3/2$, and $p = 1$. On the other hand, the 1^{st} **Layer** refers to the scenario in which the firm will either choose to refinance $CB^c_{T/2,T}$ early and simultaneously roll over the $SB^c_{T/2}$ through the proceeds from raising $CB^c_{T,3T/2}$ and SB^c_T (i.e., transfer from the first layer to the second), roll over $SB^c_{T/2}$ alone through the proceeds from raising the SB^c_T (i.e., stay in the first layer), or declare default. The values of $E^c_{T/2,T}$, $SB^c_{T/2}$, and $CB^c_{T/2,T}$ can be expressed by Equations (40), (25), and (38), in which $a = 1/2$, $b = 1$, and $p = 1/2$.

Stage 8: $0 < t < T/2$

As illustrated in Figure A.1, the first layer refers to the debt structure comprised of a $T/2$-year $SB^c_{T/2}$ and a T-year $CB^c_{T/2,T}$. The values of the equity $E^c_{T/2,T}$ and the two bonds are expressed as Equations (31), (33), and (35), in which $a = 1/2$, $b = 1$, and $p = 1/2$.

Stage 9: $t = 0$

When the firm issues $SB^c_{T/2}$ and $CB^c_{T/2,T}$ at $t = 0$, the equity value on this issue date, considering all of the debt refinancing described, is determined using backward induction in the 1rd layer black CRR tree, as described by Equation (32). The values of $SB^c_{T/2}$ and $CB^c_{T/2,T}$ can also be found using expressions of Equations (34) and (36), in which $a = 1/2$, $b = 1$, and $p = 1/2$. In this paper, we let $E^c(V_0, 0) \equiv E^c_{T/2,T}(V_0, 0)$, $SB^c(V_0, 0 \,|\, T/2) \equiv SB^c_{T/2}(V_0, 0 \,|\, T/2)$, and $CB^c(V_0, 0 \,|\, T/2, T) \equiv CB^c_{T/2,T}(V_0, 0 \,|\, T/2, T)$, as expressed in Equation (2).

This backward induction procedure enables us to find the levered equity value when the debt structure includes a non-callable bond and a longer-term callable bond with a single specified call date. Any extension of the backward-recursive pricing algorithm using a forest will be made based on this baseline scenario.

A.1.2 Extension

In this paper, a T/m-year non-callable bond SB^c, a T-year callable bond CB^c, and the corresponding equity described in Equation (2) are simultaneously evaluated under the settings of the lumpy debt maturity and the constant book leverage policy. To associate the evaluation with all possible debt refinancing activities, we employ a forest composed of several layers of CRR binomial trees. The forest in Panel C of Figure 2 addresses the scenario with $m = 2$ and the CB^c having a single call date. We make three extensions to facilitate our analysis.

The first extension is from $m = 2$ into $m > 2$ to shorten the stated maturity of SB^c and increase the firm's rollover frequency. This extension does not increase the number of forest layers, yet adds the number of debt structure statuses to individual trees for simulating additional rollover cycles. In Figure A.1, there are two debt structure statues within T years (before and after the SB^c is rolled over) in the first to third layer when m is equal to 2. If m is increased to 4, then the status will also increase to 4. In addition, the increment in debt structure statuses will increase the number of backward induction stages in the forest. For example, if m is increased from 2 to 4 within a $2T$-year time span, the backward induction procedure will be separated from 9 (i.e., $2 \times (2 \times \mathbf{2}) + 1$) in Figure A.1 to 17 (i.e., $2 \times (2 \times \mathbf{4}) + 1$) stages.

The second extension is from the evaluation framework of a $2T$-year time span into that of a NT-year time span, in which N is sufficiently large to approximate the infinite time horizon adopted by most structural credit risk models. This extension can be handled simply by adding more tree layers to a forest. The forest in Panel C is composed of 4 (i.e., $\mathbf{2} \times 2$) tree layers when N is equal to 2. If N is increased to 3 and all other things remain unchanged, then the number of tree layers will increase to 6 (i.e., $\mathbf{3} \times 2$).

The third extension is from a single call date into multiple call dates during the predetermined call protection period, which we address in two main steps. First, we start by considering a T-year callable bond with multiple redemption dates (i.e., call dates plus the stated maturity date) spaced equally apart. The bond is regarded as continuously callable when the interval between any two consecutive redemption dates matches the time step length, Δt, in the CRR tree. To illustrate, we now consider an otherwise identical callable bond $CB^c_{T/2,\,T}$ with three call dates at $t = T/4$, $T/2$, and $3T/4$, and all other things remain unchanged. The backward induction procedure for pricing equity and bonds shifts from Figure A.1 to Figure A.2. With the increase of redemption dates from 2 to 4 for the T-year bond, the number of backward induction stages within a $2T$-year time span rises from 9 (i.e., $2 \times (2 \times \mathbf{2}) + 1$) to 17 (i.e., $2 \times (2 \times \mathbf{4}) + 1$), and the number of tree layers within the forest grows from 4 (i.e., $2 \times \mathbf{2}$) to 8 (i.e., $2 \times \mathbf{4}$).[30] Each backward induction stage's evaluation procedure mirrors the details introduced in Appendix A.1.1. For example, the procedures in Stages 9 and 11 in Figure A.2 are similar to those in Stage 5 in Figure A.1; those in Stages 10 and 12 in the former figure are identical to those in Stage 4 in the latter.

Since equity and bonds are evaluated using a finite NT-year time span, we also notice that callable bonds cannot always be refinanced through proceeds from raising otherwise identical ones. In the case of $N = 2$ in Figure A.2, bonds issued at (after) $t = 5T/4$ (i.e., in the sixth to eighth layers) are not otherwise identical to previously-issued T-year callable bonds; only the coupon rates and face values are identical. For example, the callable bond in the sixth layer, $CB^{c*}_{3T/2,\,2T}$, has a stated maturity of $3T/4$ years rather than a T-year since its maturity date should be at $t = 2T$. In addition, the callable bond $CB^c_{T,\,7T/4}$ in the fourth layer will be refinanced on its stated maturity date $t = 7T/4$ with the $T/4$-year non-callable SB^{s*}_{2T} due to the same reason. The robustness check displayed

[30] More generally, when we use an NT-year forest to evaluate a T-year callable bond with L redemption dates, and all other things remain unchanged, the number of backward induction stages equals $N \times (2 \times L) + 1$, while the number of tree layers equals $N \times L$.

in Table A.2 in Appendix A.1.3 shows that the impact of this ad hoc setting on the initial values of $SB^c_{T/2}$ and $CB^c_{T/2,T}$ is trivial when N is great enough.

Second, we focus on evaluating callable bonds with multiple call dates within a set call period. When a callable bond has only one permissible call date, we can evaluate a levered firm with this bond as well as a shorter-term non-callable bond using the backward induction procedure depicted in Figure A.1. By removing the call date and replacing the evaluation procedures in Stage 3 (7) with those in Stage 2 (6) (i.e., omitting the second and fourth tree layers), we can also evaluate an otherwise identical firm that issues two non-callable bonds with different maturities. Similarly, for callable bonds with multiple call dates, we can establish call protection periods of various lengths by eliminating earlier call dates. For instance, in Figure A.2, a $T/2$-year call protection period is provided by removing the call dates marked as “A”. When backward induction reaches the first layer tree, the evaluation procedures in Stage 15 are simplified to those in Stage 14 within the same layer. This simplification also appears from the second to sixth layer if the callable bond is refinanced with an identical callable. Additionally, if call dates labeled as "B" are also removed, call protection period extends to $3T/4$ years, with further reduction in evaluation procedures in Stage 13 to those in Stage 12 in the first layer, and similar reductions applied to other layers. Overall, this method effectively adapts the backward-recursive pricing algorithm to accommodate callable bonds with varying call dates and protection periods, thus providing a robust framework for assessing their impact on the total value of a levered firm.

A.1.3 Robustness Checks

Before applying our quantitative framework, it is important to verify that our framework generates accurate and stable pricing results. Prior studies such as Broadie and Kaya (2007) and Wang et al. (2014) examine the robustness of their tree methods by showing that pricing results converge to analytical solutions with the increment in the number of time steps (Duffie, 1996). However, analytical pricing formulas are not available for a finite-maturity callable bond.

Instead of directly confirming the correctness of pricing results, we follow Liu et al. (2016) and indirectly check the rationality of pricing results as a whole by employing capital structure irrelevance theory proposed by Modigliani and Miller (1958). Essentially, in the capital market without frictions (i.e., no corporate income taxes, bankruptcy costs, and debt flotation costs), the market value of a firm is independent of its capital structure. As a result, a levered firm value $V^{L.c}$ in Equation (2) generated by our proposed framework should be equal to the unlevered firm value V in Equation (1) under otherwise identical conditions. Our numerical results in Panel A of Table A.1 show that the initial levered firm value $V_0^{L.c}$, which is equal to the lump sum of SB^c and CB^c plus corresponding equity values, is equal to the initial unlevered firm value V_0 (100 in Panel A) regardless of the debt structure scenarios listed in the first row. Under the equality, we observe that yield spreads of P-year SB^c increase with P, since credit risk increases as the stated maturity increases. Given that the stated maturity of CB^c is 10 years, the yield spreads of CB^c decrease with increments in the call protection length P, because CB^c holders are granted more protection against call risk and thus require lower call risk premiums. We also observe that yield spreads of the SB^c converge to those of CB^c as P is equal to 10 years, since the callable degenerates into SB^c as its call protection length equals its stated maturity. These observations persist even when the capital structure irrelevance theory is invalid, as exhibited in Panel B. On the other hand, equity and bond values generated by our framework are stable. We observe that all pricing results in Table A.1 change little (i.e., less than 0.2% for equity values and 5 basis points for bond yield spreads) when `Time steps` (listed in the second row) increases from 32 to 128.

Since equity and bonds are evaluated using a finite time span equal to $N \times 10$ years, we note that bonds cannot always be refinanced by issuing otherwise identical ones, as mentioned in Appendix A.1.2. We now check how the value of N and the aforementioned ad hoc setting influence the pricing results of the P-year SB^c and the T-year CB^c. We observe in Table A.2 that the value of N has no impact on all pricing results when capital markets are frictionless in Panel A, and has little impact on bond prices when capital markets have frictions in Panel B. *Ceteris paribus*, an increase in the entire time span will enhance equity values, and higher equity values will delay the firm's default decision, thereby decreasing bond yield spreads. However, the impact of this delay on the prices of finite-maturity bonds is limited. In addition, the ad hoc scenario will only appear at a time far away from the life of our targeted bonds if N is great enough. Therefore, the impact of this scenario is also limited. As shown in Panel B, the yield spreads of the two bonds decrease by less than 3 basis points as the N listed in the second row increases from 5 to 7.

A.2 Interpretation of Data for First Call Dates and Action Effective Dates

We retrieve first call dates from call schedules, refund protection, and make-whole call provisions. First, if a bond's entire call schedule, named "CALL_SCHEDULE" in Mergent FISD, is available, we set its first call date to the earliest call date in the schedule. If this isn't the case but the bond has a complete call schedule available in Bloomberg, we select the earliest call date as the bond's first call date. If the aforementioned two data sources are absent, we set the bond's first call date to the CALL_DATE recorded in SDC if available. Here we utilize nine-digit issue CUSIPs as bond identifiers across the aforementioned three databases. Next, if the call schedule is unavailable, then we detect the presence of the refund protection (i.e., REFUND_PROTECTION = "Y" in Mergent FISD) and use the REFUNDING_DATE as its first call date as in Powers (2021). Third, for a callable bond that has a make-whole call provision but lacks a call schedule and refund protection, we set its first call date to coincide with its make-whole start date, named"MAKE_WHOLE_START_DATE" in Mergent FISD. Finally, we utilize a callable bond's "INITIAL_CALL_DATA" in Mergent FISD if the three pieces of information that we have discussed are unavailable. We pick the date following "NC", which represents "**N**ot **C**allable until", as the first call date. For example, the first call date is set to "10/16/2015" for the callable bond with recorded information"NC 10/16/2015 CONT @ PAR." On the other hand, if the recorded information starts with "CC", which denotes **C**ontinuously **C**allable, then the first call date is set to the bond offering date.

The EFFECTIVE_DATE in Mergent FISD is the action effective date corresponding to the ACTION_TYPE for a bond. In this paper, we merely focus on seven ACTION_TYPEs that indeed lead to changes in outstanding amounts, which are coded as "B," "E," "P," "IRP," "T," "F," and "IM" in Mergent FISD. The action with ACTION_TYPE = "IM" in Mergent FISD refers to redemption at maturity. The ACTION_TYPE = "B," "E," "P," "IRP," "T," or "F" refers to redemption before maturity, in which the former three types indicate redemption through calls, "IRP" indicates redemption through repurchases, "T" indicates tender offers, and "F" indicates refunding. The effective maturity of a bond, denoted by *BondEffM*, is defined as a simple average of the time span (measured in years) between the bond issue date and each redemption effective date. If all outstanding amounts are redeemed at once, then the bond's *BondEffM* is set to the time span between the issue date and the unique redemption effective date.

A.3 Mapping Mergent FISD with Compustat

We merge the bond data in Mergent FISD with the firm data in Compustat to build our firm-level sample. Since the unique firm identifier in Compustat, GVKEY, is not recorded in Mergent FISD, we employ CIK and six-digit issuer CUSIP as bridges to match Mergent FISD bond issuers to Compustat and SDC. To make our firm-level data more complete, we hand collect data from the U.S. Security and Exchange Commission (SEC)[31] to supplement some missing data. Since some bond issuers' information in Mergent FISD

[31] EDGAR database

are different or missing in Compustat due to reasons like corporate restructuring, we collect the information from SEC and other public finance information platforms like Bloomberg to replace and fill.

A.4 Variable Definitions

The variables used in our empirical analysis are listed in Table A.3. The bond-level variables in Panel A are defined mainly according to the data elements in Mergent FISD. In particular, a bond may have ratings from four different credit rating agencies. To determine the rating on the bond issue date, we prioritize S&P's bond rating (abbreviated as SPR). If the SPR is missing, we turn to Moody's (MR). If both the SPR and MR are unavailable, we then consider Fitch's (FR), followed by Duff and Phelps's (DPR). We note that Mergent FISD transforms bond ratings from the four credit rating agencies into ordinal scores, and we follow the scores assigned by Mergent FISD. For example, it assigns Aaa (AAA) from Moody's (S&P, Fitch, and Duff and Phelps) to 1, and Ca (CC) from Moody's (S&P and Fitch) to 21.[32] The firm-level control variables used in the regression are defined in Panel B, mainly according to the data elements in Compustat. To mitigate the impact of outliers and possible coding errors, we winsorize all firm-level variables at the upper and lower one percentiles. This winsorization is consistently applied across all of our analyses.

Our firm-level dependent variables, *FirmCoupon*, *FirmCProtR*, and *FirmElimR*, are defined in Equations (16), (17), and (18). They are constructed using the bond-level variables in Panel A of Table A.3 to characterize the bond characteristics at the firm level. To illustrate how we construct these variables, we provide a simple example in Table A.4. We consider a firm (i.e., i-th firm) with two callable bonds outstanding during the period 1998-2009, Callable Bond 1 and Callable Bond 2. For ease of illustration, we assume that

[32] We observe alignment in the first 16 credit ratings, ranging from Aaa (AAA) to B3 (B-) provided by Moody's (S&P, Fitch, and Duff and Phelps). For the first 5 ratings below B3 (B-), Moody's Caa1 to C align with S&P's and Fitch's CCC to C. Below the C rating, Moody's only has two rating categories, while S&P's and Fitch's both have four.

the firm just redeemed a previously-issued bond through proceeds from raising Callable Bond 1 in 1998. Since this action is an early refinancing activity as defined in Xu (2018), we use a binary variable $D(EarlyRefinancing)_{i,t,j}$ to mark the year of this refinancing date. For Callable Bond 1, the date is in the year 1998, so $D(EarlyRefinancing)_{i,t,1}$ should be assigned a value of one when subscript t equals 1998 and zero when the t equals other years in the bond's lifespan, as exhibited in Panel A. The callable has a coupon rate of 7%, a 10-year stated maturity, and a 5-year call protection period. We then use another binary variable $D(TurnCallable)_{i,t,j}$ to mark the year of the bond's first call date. In this case, the first call date is in the year 2003, so $D(TurnCallable)_{i,t,1}$ should be assigned a value of one when subscript t equals 2003 and zero when the t equals other years in the bond's lifespan. Based on these settings, we further suppose that Callable Bond 1 was redeemed early right on its first call date through the firm's own internal funds rather than the proceeds from raising new bonds. Since Callable Bond 2 was issued in 2001 not for early refinancing of an outstanding bond, $D(EarlyRefinancing)_{i,t,2}$ equals zero for all t in the bond's lifespan. In addition, the callable has a coupon rate of 4.8%, a 8-year stated maturity, and a 3-year call protection period. Thus, $D(TurnCallable)_{i,t,1}$ should be assigned a value of one when the subscript t equals 2004 and zero when the t equals other years of the bond's life. Finally, it should be redeemed early right on its first call date in the year 2004 through the firm's own internal funds. Other information is detailed in Panel B.

The firm-level variables in Panel C are constructed using the bond-level variables in Panels A and B. In particular, since *BondStaM* (*BondCProt*) is the stated maturity (call protection length) determined at the time of debt issuance, it stays constant before the year in which the bond is retired.[33] For example, Callable Bond 1 exhibits the *BondStaM* of 10 during the period between 1998 and 2002. On the other hand, *BondElim* is computed

[33]We calculate the two-date time span in years with precision in terms of days in our empirical analysis. However, for clarity's sake, we present the calculated figure with the unit of years here in Table A.4. The same rule is applied to *BondElim*.

as the time span in years between a bond's redemption effective date and stated maturity date; it is defined only in the year in which the bond is retired. For example, Callable Bond 1 exhibits the *BondElim* of 5 only in the year of 2003. *BondCoupon* is the nominal coupon rate of each bond. Similar to *BondStaM* and *BondCProt*, it is defined during the period between debt issue and retired years. While most coupon rates for our bond sample are fixed-type, we trace the changes in coupon rates for the reset bonds and adjust their coupon rates annually before the retired year. In Panel C, $D(TurnCallable)_{i,t}$ and $D(EarlyRefinancing)_{i,t}$ are the firm-level binary variables. Their values in one year equals one if there is at least one bond turns callable (or been early refinanced) in that year. For all other firm-level variables, they are calculated as the simple averages of the bond-level variables in Panels A and B in the corresponding time location by following the definitions in Equations (15), (16), (17), and (18).

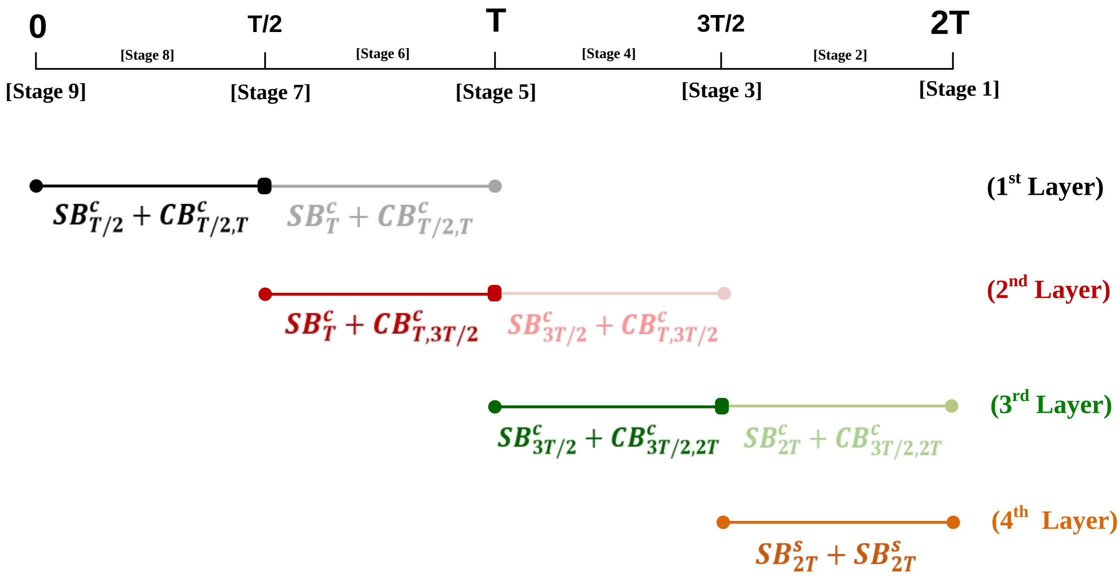


Figure A.1: Nine stages for the backward induction procedure when applying the forest in Panel C of Figure 2. In this case, every T-year callable bond CB^c has only one permissible call date, which occurs at the midpoint of a bond's lifespan and is represented by a rectangle in this figure. All non-callable bonds, SB^c and SB^s, have a stated maturity of $T/2$ years.

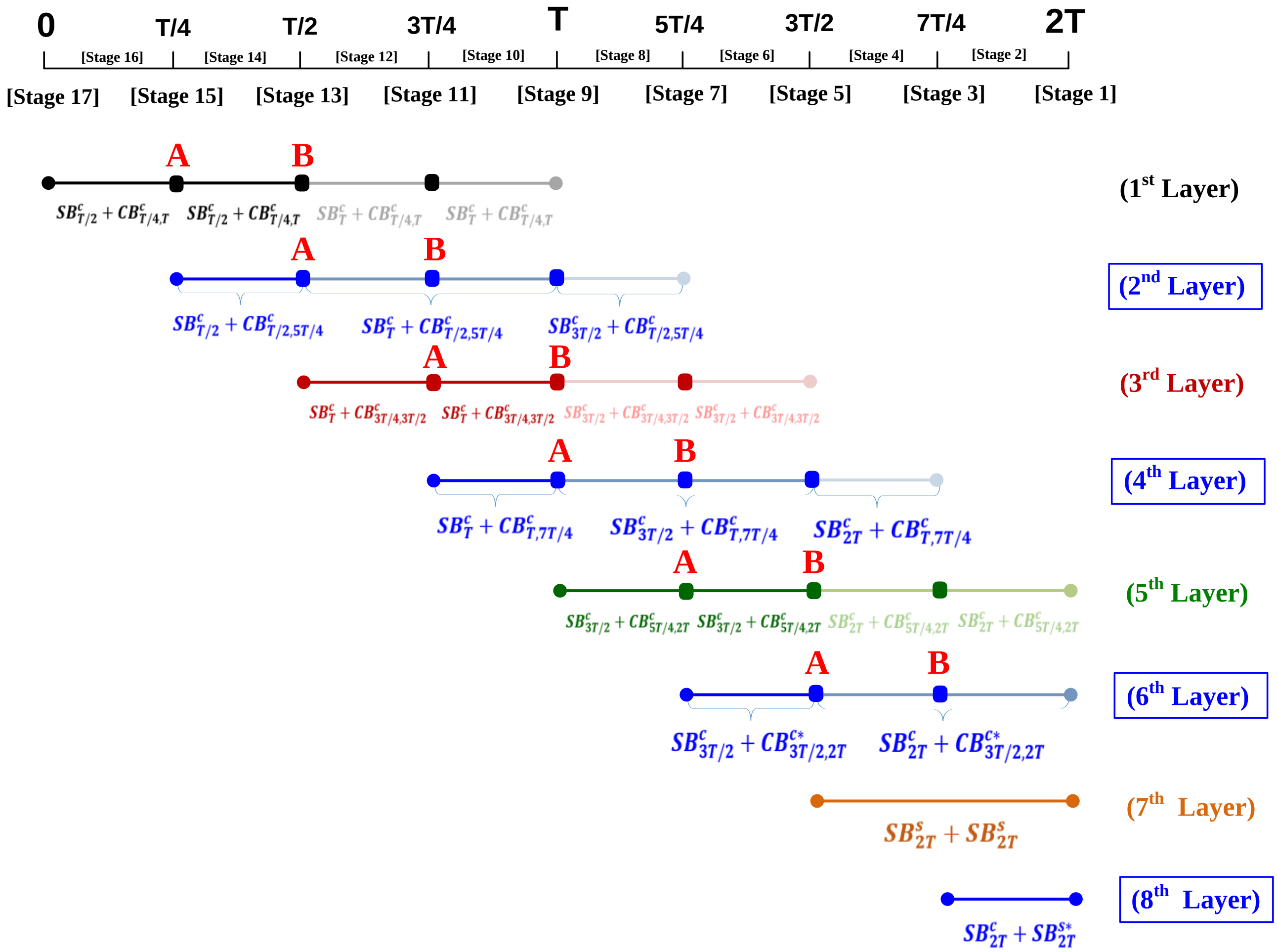


Figure A.2: Backward induction procedure for evaluating a callable bond with three call dates. In this case, each T-year callable bond CB^c has three allowable call dates, which are spaced equally apart and are represented by rectangles. The callable bond appeared in the sixth layer, CB^{c*}, has a stated maturity of $3T/4$ years and two call dates. All non-callable bonds, SB^c and SB^s, have a stated maturity of $T/2$ years, except for the one: SB^{s*} in the eighth layer, which has a stated maturity of $T/4$ years.

Table A.1: Convergence of equity and bond values.

We consider a firm with the debt structure composed of a P-year non-callable bond SB^c and a 10-year callable bond CB^c with a P-year call protection period. Bond prices, corresponding bond yield spreads in basis points (listed in parentheses), and equity values E^c (in the last row) generated by our numerical framework are examined under three different scenarios denoted by the first row. `Time steps` in the second row denotes the number of time steps used to partition the 1-year time span in our framework. The framework is built using a 60-year time span. The total debt face value is 61.68; the face values for SB^c and CB^c account for 8% and 92% of the total debt value, respectively. The coupon rates for the two bonds are all set to 6%. CB^c can be refinanced early through a call once per year after the call protection period expires. The scheduled call prices are all set to the face value of CB^c. The firm's prevailing asset value V_0 is 100, its volatility σ is 21%, and the risk-free rate r is 4.61%. Panel A displays the pricing results when the corporate tax rate τ, bankruptcy cost ω, and debt flotation cost γ are all set to 0. Panel B displays the results when the τ, ω, and γ are set to 30.6%, 37%, and 0.5%, respectively.

Panel A: Capital market without frictions

Scenario	$P=1$			$P=5$			$P=10$		
`Time steps`	**32**	**64**	**128**	**32**	**64**	**128**	**32**	**64**	**128**
$SB^c(P\text{-yr})$	5.00	5.00	5.00	5.12	5.13	5.12	5.20	5.20	5.20
	(1.83)	(1.51)	(1.76)	(52.30)	(51.09)	(51.76)	(68.48)	(68.30)	(68.25)
$CB^c(10\text{-yr})$	57.07	57.06	57.07	56.70	56.72	56.70	58.89	59.89	59.86
	(130.87)	(131.32)	(131.28)	(102.26)	(102.26)	(102.24)	(68.48)	(68.30)	(68.25)
E^c	37.93	37.94	37.93	36.56	36.54	36.56	34.97	34.95	34.95

Panel B: Capital market with frictions

Scenario	$P=1$			$P=5$			$P=10$		
`Time steps`	**32**	**64**	**128**	**32**	**64**	**128**	**32**	**64**	**128**
$SB^c(P\text{-yr})$	5.00	5.00	5.00	5.12	5.12	5.12	5.06	5.05	5.06
	(0.28)	(0.56)	(0.34)	(54.69)	(50.68)	(52.30)	(106.39)	(106.92)	(108.24)
$CB^c(10\text{-yr})$	55.34	55.34	55.29	56.70	56.72	56.70	58.13	58.12	58.13
	(171.86)	(172.12)	(173.60)	(139.52)	(139.32)	(139.95)	(106.39)	(106.92)	(108.24)
E^c	49.16	49.12	49.09	51.21	51.21	51.22	50.53	50.49	50.49

Table A.2: Robustness check for equity and bond values priced by the quantitative framework with different time spans.

Bond prices, corresponding bond yield spreads in basis points (listed in parentheses), and equity values E^c (in the last row) generated by our quantitative framework are examined under three different scenarios denoted by the first row. N in the second row represents that our quantitative framework is constructed using a time span equal to $N \times 10$ years. The number of time steps to partition the 1-year time span is set to 32. All other settings mirror those in Table A.1.

Panel A: Capital market without frictions

Scenario	$P=1$			$P=5$			$P=10$		
N	**5**	**6**	**7**	**5**	**6**	**7**	**5**	**6**	**7**
$SB^c(P\text{-yr})$	5.00	5.00	5.00	5.12	5.12	5.12	5.20	5.20	5.20
	(1.83)	(1.83)	(1.83)	(52.30)	(52.30)	(52.30)	(68.48)	(68.48)	(68.48)
CB^c (10-yr)	57.07	57.07	57.07	58.32	58.32	58.32	59.83	59.83	59.83
	(130.87)	(130.87)	(130.87)	(102.26)	(102.26)	(102.26)	(68.48)	(68.48)	(68.48)
E^c	37.93	37.93	37.93	36.56	36.56	36.56	34.97	34.97	34.97

Panel B: Capital market with frictions

Scenario	$P=1$			$P=5$			$P=10$		
N	**5**	**6**	**7**	**5**	**6**	**7**	**5**	**6**	**7**
SB^c (P-yr)	5.00	5.00	5.00	5.12	5.12	5.12	5.05	5.06	5.06
	(0.28)	(0.28)	(0.28)	(55.25)	(54.69)	(54.19)	(107.43)	(106.39)	(105.84)
CB^c (10-yr)	55.28	55.34	55.35	56.63	56.70	56.73	58.09	58.13	58.16
	(173.21)	(171.86)	(171.32)	(141.18)	(139.52)	(138.84)	(107.43)	(106.39)	(105.84)
E^c	48.81	49.16	49.36	50.70	51.21	51.53	50.04	50.53	50.82

Table A.3: Variable Definitions

This table provides the construction of our bond-level and firm-level variables used in our empirical analysis. *RI* denotes refinancing intensity. *Curlia* denotes current liability. *M/B Ratio* denotes market-to-book ratio.

Variable	Definition
Panel A: Bond-level variable	
BondStaM	Time span in years between a bond's offering date and stated maturity date
BondEffM	Time span in years between a bond's offering date and redemption effective date
BondCProt	Time span in years between a bond's offering date and first call date
BondCProtR	*BondCProt/BondStaM*
BondElim	Time span in years between a bond's redemption effective date and stated maturity date
BondElimR	*BondElim/BondStaM*
BondCoupon	The nominal coupon rate of a bond
Covenant count	Number of restrictive covenants present in one bond; the restrictive covenants are identified according to the definition in Billett et al. (2007).
Bond rating	The ordinal rating score assigned by Mergent FISD. The priority of the rating selection: SPR $\succ$ MR $\succ$ FR $\succ$ DPR .
Panel B: Firm-level control variable	
Leverage	Total assets/Total stockholders' equity, as in Kalemli-Ozcan et al. (2012).
RI	Long-term debt due in one year/(Long-term debt due in one year + Debt due in more than one year)
Curlia	Debt in current liability/(Debt in current liability + Debt due in more than one year) as in Duchin et al. (2010)
ln(Asset)	Natural log of total assets
M/B Ratio	(Total assets−Common equity+Common shares outstanding×Closing price (fiscal year))/Total assets
Tangibility	Property, plant and equipment/Total assets
EBITDA	Earnings before interest, tax, depreciation and amortization/Total assets
Cash	Cash and short-term investment/Total assets
Equity return	ΔClosing price (fiscal year)/L.Closing price (fiscal year), adjusted for cumulative adjustment factor if applicable
TermSpread	Difference between the 10-year and 1-year corporate yield according to the data items in Federal Reserve Board's H.15 Report
Firm rating	The ordinal score of S&P long-term firm credit rating; AAA = 1, AA+ = 2,..., and D=22, as in Gopalan et al. (2014)

Table A.4: Constructing Firm-level Variables from Bond-Level Variables

This table provides a scenario for illustrating the construction of firm-level variables from bond-level variables. We consider a firm with two callable bonds outstanding during 1998-2009. "Callable Bond 1" was issued in the year 1998 to refinance a previously-issued bond, as shown in Panel A. The callable has the coupon rate of 7%, 10-year stated maturity, and 5-year call protection period, and it was finally redeemed on its first call date in 2003 through the firm's internal funds. "Callable bond 2" was issued in the year 2001, as shown in Panel B. The callable has the coupon rate of 4.8%, 8-year stated maturity, and 3-year call protection period, and it was finally redeemed on its first call date in 2004, also through the firm's internal funds. The firm-level data in Panel C are calculated using bond-level variables in Panels A and B.

Panel A : Callable Bond 1	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
$D(TurnCallable)_{i.t,1}$	0	0	0	0	0	1	0	0	0	0	0	
$D(EarlyRefinance)_{i,t,1}$	1	0	0	0	0	0	0	0	0	0	0	
$BondStaM_{i,t,1}$ (yrs)	10	10	10	10	10							
$BondElim_{i,t,1}$ (yrs)						5						
$BondElimR_{i,t,1}$						0.5						
$BondCProt_{i,t,1}$(yrs)	5	5	5	5	5							
$BondCProtR_{i,t,1}$	0.5	0.5	0.5	0.5	0.5							
$BondCoupon_{i,t,1}$ (%)	7	7	7	7	7							
Panel B : Callable Bond 2	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
$D(TurnCallable)_{i.t,2}$				0	0	0	1	0	0	0	0	0
$D(EarlyRefinance)_{i,t,2}$				0	0	0	0	0	0	0	0	0
$BondStaM_{i,t,2}$ (yrs)				8	8	8						
$BondElim_{i,t,2}$ (yrs)							5					
$BondElimR_{i,t,2}$							0.625					
$BondCProt_{i,t,2}$(yrs)				3	3	3						
$BondCProtR_{i,t,2}$				0.375	0.375	0.375						
$BondCoupon_{i,t,2}$ (%)				4.8	4.8	4.8						
Panel C : Firm Level	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
$D(TurnCallable)_{i.t}$	0	0	0	0	0	1	1	0	0	0	0	0
$D(EarlyRefinance)_{i,t}$	1	0	0	0	0	0	0	0	0	0	0	0
$FirmStaM_{i,t}$ (yrs)	10	10	10	9	9	8						
$FirmElim_{i,t}$ (yrs)						5	5					
$FirmElimR_{i,t}$						0.5	0.625					
$FirmCProt_{i,t}$ (yrs)	5	5	5	4	4	3						
$FirmCProtR_{i,t}$	0.5	0.5	0.5	0.4375	0.4375	0.375						
$FirmCoupon_{i,t}$ (%)	7	7	7	5.9	5.9	4.8						